DETERMINATION OF OPTIMUM WATS LINE MIX:
A SIMULATION STUDY

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Elizabeth Stansell Tice

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DETERMINATION OF OPTIMUM WATS LINE MIX:

A SIMULATION STUDY

Approved:

William W. Hines, Chairman

Robert B. Cooper

D. C. Montgomery

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SUMMARY

An electronics firm based in Florida utilizes an extensive system of telecommunications of which an integral part is the Wide Area Telephone Service. WATS is a system offered by the Telephone Company which allows direct access to toll (long distance) facilities through leased lines, rated by the distance or zones they cover. The purpose of this study is to investigate the system and mix of WATS lines to find if another mix of lines might provide the same or better service at a reduced cost to the company.

Access to the WATS lines is made by placing a call request with a switchboard operator. If the proper zone line is not available, the call request is placed in queue before the desired zone lines on a first-in-first-out basis. A systems usage policy was introduced which stated that a call request would be held in queue until (1) a proper line became available, or (2) the waiting time had reached a specified maximum delay time. In the second case a search is made for any available higher ranked WATS line before the call is placed over regular toll facilities. Total cost of the WATS system is the sum of the lease costs of the various lines plus the cost of the toll calls overflowing the WATS system.

Queueing and Teletraffic theory have been used to analyze various situations involved in this problem. P. J. Burke (unpublished) derived formulas to calculate the costs of different mixes of full and measured rate WATS lines by using the principles of overflow and the Erlang Loss Formula. However, due to the ranking of the WATS lines and

the maximum delay time policy, it was determined that simulation of the system would be the best approach to a solution.

Data used to simulate call arrival patterns and holding times on calls were obtained from the company under study. Distribution of both the inter-arrival times and holding times was exponential. The simulation was written in GPSS II and run on a UNIVAC 1108 computer. Each run simulated three days (8 a.m.-5 p.m.) of operation. Seven different WATS line mixes were tested against two maximum delay time levels and the number of toll calls generated by each combination was used to compute the associated monthly toll costs. The simulation program also printed out data on average time in queue, average utilization of the WATS lines, number of pending call requests, and other information useful in evaluating the level of service provided by each line mix.

The results of the study show that line mix D (022304) best meets the requirements of minimal cost of operation while maintaining an acceptable level of service. Total monthly cost of Mix D with maximum delay time of 15 minutes is estimated at $21,904.00. Toll costs associated with the 20-minute maximum delay time were projected to be $554.00 per month lower than the MDT of 15 minutes, while the difference in average waiting time was less than two minutes. Study of the system over a period of time revealed a changing pattern of calling habits which further supports the recommendation of periodic analysis of the WATS system.

CHAPTER I

INTRODUCTION

Statement of the Objectives

A Florida-based electronics firm utilizes an extensive telecommunications system for business contacts throughout the United States. An important part of this firm's telecommunications network is the Wide Area Telephone Service (WATS) leased from Southern Bell Telephone and Telegraph Company. Full WATS provides what might be considered a package plan for long distance service. For a fixed charge per month, unlimited long distance calls can be made within a designated area as well as in all lower rated areas. The United States is divided into six somewhat concentric zones specified by their distance from the source, and containing roughly the same number of telephones. For instance, the calling area, from Florida, for Zone 1 includes seven southeastern states (Louisiana, Mississippi, Alabama, Georgia, North Carolina, South Carolina, Tennessee), while a Zone 6 line can access any state in the country (see Figure 1).

The mix of WATS lines (six Zone 6 and five Zone 4) presently leased by the company and its system of usage is not considered optimal for either economy or service. The objective of this study, therefore, is to determine a *good*[†] mix of WATS lines for the company, and also to

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[†]A *good* mix is one in which cost is held to an acceptable level while at the same time service performance for the user is high.

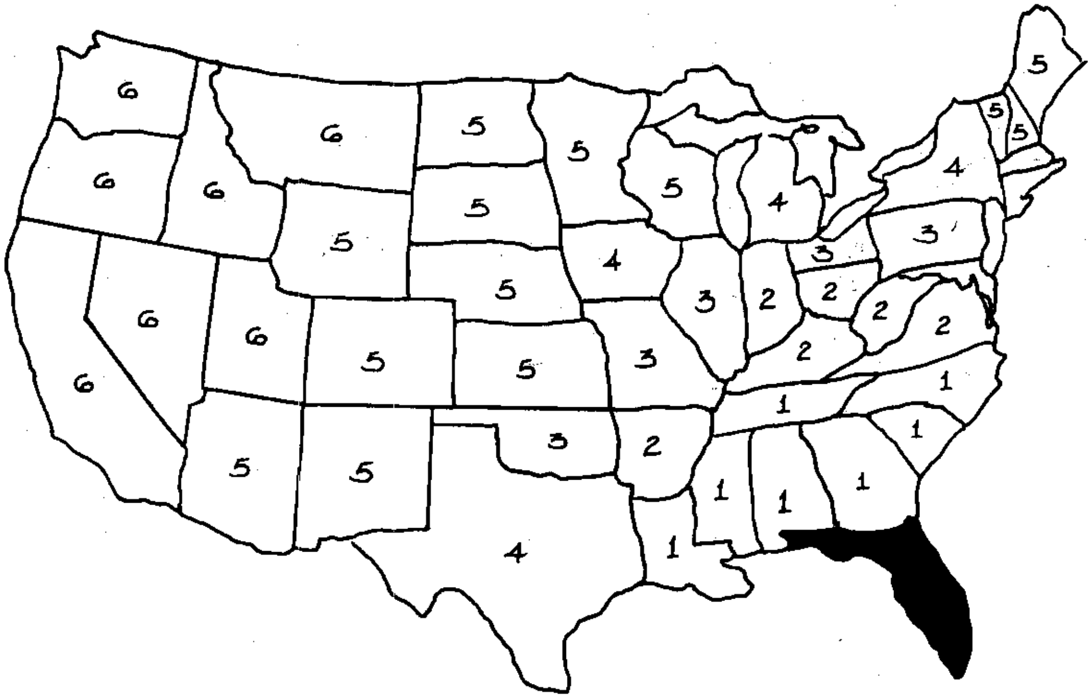

Figure 1. Interstate WATS Zones for the State of Florida

recommend a systematic approach to efficient access and usage policies for these lines.

Importance of the Problem

In this modern era of rapidly expanding communications facilities and networks, keeping up with communications progress and available services has become essential for companies that rely on customer contact. The telephone call has all but replaced the traditional letter as a means of communication, particularly when speed of delivery is important. Further, while the cost of a business letter is rising, long distance telephone service costs are dropping. Dartnell Institute of Business Research has placed the cost of a business letter in 1972 at \$3.20, showing an increase of nearly 75 per cent since 1960, when the cost was \$1.83 per letter.[†] During this same period (1960-1972) the cost of long distance service has decreased approximately 10 per cent.[††] The advantages of instant long distance telephone communication have become even greater due to computer data transmission and development of methods to reproduce printed material over a telephone hook-up.

The company under study has been very conscious of its need for an effective telecommunications system. To this end, they have stressed serviceability of their system above cost, and presently they are

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[†] Southern Bell Notes, July, 1972.

[††] Mr. Lawrence H. Brice of the Southern Bell Telephone Company (Atlanta Area Office) reported that "since 1960, the dollar amount of revenue in Georgia for intrastate long distance calls is approximately 10 per cent less than it would be if there had been no rate decrease. This same percentage applies generally to interstate toll call revenue also" (9/7/72).

spending in excess of $1,000,000 a year for telephone service. Their WATS system alone costs over $21,000 a month for the six Zone 6 and five Zone 4 unlimited usage lines.

Not only is it important for today's companies to have good, up-to-date communications systems--they must also keep a constant eye on capital outlay. The cost of expansion has been given major attention in recent years due to the cost of money. Too often a company has concerned itself with the cost of expansion and overlooked the method of expansion. As a result many companies have followed a pattern of "add-on" growth: when expansion became necessary, the existing facilities were simply "added-on-to." Unfortunately, the inadequacies of an original scheme will, in many cases, be amplified if it is simply enlarged, and the consequent inefficiency means lost dollars.

In regard to the problem at hand, it appears that the number of interstate WATS lines leased by the company has been increased in an "add-on" fashion to take care of extra calling volume and to reduce waiting time for a line, without extensive study of the WATS system. Studies of the existing telecommunications network must be initiated by the company itself since the Telephone Company has no analytical method for up-dating an existing WATS system. The only computer program the Telephone Company (Southern Bell) uses in connection with WATS service is one which determines expected volume of traffic for Inward WATS lines. (As implied by the name, Inward WATS service provides toll-free long distance service to those persons dialing *into* the system. An 800 Area Code indicates Inward WATS service.) Even this program is limited to analyzing data for a proposed system, and is used only as a precaution against overloading trunking equipment in a Central Office.[†] As for Outward WATS, which is the concern of this study, there is no computer program or analytical method used by the Telephone Company to determine the best number and mix of lines. Suggestion of a suitable WATS mix is a judgment decision done on a rule-of-thumb basis. It includes no analytical review of call volume to particular areas, interarrival times, company policies on delay times, etc., and how these factors interact.

In order to treat this system, guidelines for usage of the WATS lines had to be predetermined. The company under study has set up policies on call priorities and procedures for completing calls, but, for example, a specific mandatory delay time before the call can go out on a toll line, which would help regulate the number of toll calls, has never been initiated. This has caused both excessive delay, for those without sufficient priority, and premature toll calls, neither indicative of an optimal system.

In conclusion, the importance of this problem lies in the fact that companies cannot overlook such "overhead" or expected costs as telephone service when dealing with expense controls, nor should they fail to analyze the systems aspects of such a costly service before simply enlarging it. In addition, system effectiveness is often determined by the company policy regulating its use. It is, therefore,

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[†]Personal communication with Mr. Don Jones, Marketing Department, Southern Bell Telephone Company, Atlanta, Georgia, 8/25/72.

important for company policy to be formulated and implemented.

Scope and Limitations

For purposes of this thesis, the scope of the problem of reducing telecommunications expense has been limited to determining the "optimal" interstate WATS line mix for the participating company. This is accomplished by considering enumerated feasible levels for line availability, usage, and cost. The model being designed for this particular firm involves a systems usage policy, defined by the author and concurred in by the company, that becomes the major limitation of this simulation. Otherwise, by changing the call rate parameters, this simulation model should be generally applicable. Briefly, the policy of systems usage states that, whenever possible, a long distance call should go out on a line of its first choice WATS zone, i.e. the lowest rated zone that will carry the call. If within a specified maximum delay time the call is not completed, the choices are to use a higher rated line, if available, or to use a toll line. In any case, the placing of the call will not be delayed longer than the maximum delay time, nor will the call be placed on a toll line before this maximum delay time is reached.[†]

This is not the exact system of usage presently employed by the company under study, and consequently the simulation is not an exact simulation of existing operations. However, it was felt that by defining a practicable policy for toll call spill-over, as is contained in

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[†]Special exception to this policy would be the estimated five calls per day that are of high priority and are placed immediately on WATS, if available, or Toll.

the simulation, there would be an increased probability of forming and testing a system of WATS communication desirable in terms of economy and service.

The standards of the recommended policy, which were suggested by Mr. C. G. Sieger, Telecommunications Coordinator of Reynolds Metals Company, are simple to follow, and they guarantee completion of a call within a specified interval. Under the priority-governed policy now in effect at the company, some calls are delayed three hours or more, while others go out on toll lines with little or no delay. The idea behind priority is to reduce delay time on a call for the priority caller. The premise underlying a maximum delay time policy is that almost any caller can afford to wait up to a reasonable maximum delay time, knowing that his call will be placed within that time. If it is urgent that the call be placed immediately, this can be done, either on WATS or on Toll lines, but the estimated[†] five such calls per day would not be sufficient to affect the general policy.

An additional advantage to be gained by using a maximum delay time (MDT) policy is a reduction of the operator time wasted in trying to locate a caller once a line is secured for his call. Knowing that his request for a long distance call will be met within a set amount of time, a caller is much less likely to be unavailable during that time, thus making the operator's job more efficient. In turn, the operators provide the means for queuing the call requests which gives rise to a

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[†]Estimate provided by management personnel as a "best opinion" estimate.

system of sequential calls. WATS is most economically effective only when requests for lines are sequential, as stated by Nordling (9).

The major decision variable being considered for its effect is the number of lines leased for particular WATS zones, that is, the "mix" of WATS lines. The program has been structured so that policies which allow (a) different specified maximum delay times and (b) a direct access capability to the WATS lines by some callers can be simulated to study their effect. The direct access feature is presently in use at the company, but if the simulation indicates that it is not necessary, or that it compromises system effectiveness, the feature can be made available only after the switchboard closes. Except for treating the direct access aspect, the intricacies of call priorities are not considered in this simulation. Also, requests that calls be completed at a particular time in the future are not included: call requests are assumed to be made at the time the callers wish to complete their calls.

In the original handling of this problem (by G. Sharp and E. Stansell) a great deal of the emphasis was placed on the role of the switchboard operator, i.e. how long she spent on each call request, who got the tickets for which calls, etc. In this study the author feels that the actual mix of the lines should not be affected by the operating procedures as long as the call requests are handled efficiently. The company feels that the service they have been receiving from these operators is satisfactory. The problem as defined does not consider the human factors of operating practices, nor does it consider varying the number of operators.

It is interesting to note that, according to AT&T material (2) on operator efficiency, the down-time (time when an operator (or line) is idle) necessary to provide for the peaks and valleys of call arrival distributions increases considerably as the operating staff grows smaller. For an operating staff of five, an average down-time of 30 per cent must be provided to insure *good* service. For much larger groups of operators, the best average percentage of efficiency peaks at 92 per cent, or 8 per cent down-time. If "efficiency" runs much higher, fatigue causes a slowdown in operator service, finally causing a loss in efficiency. The fatigue factor explains why the operating staff at the company can make up for one operator being on break or lunch (a short period of time) but cannot operate well under those conditions continuously.

The computer simulation model has been planned to be as simple and straightforward as possible. Due to the type of data gathered, greater complexity would only involve uncertainty, i.e. probabilities of persons doing this or that. Thus, it was felt that simplicity would enhance the usefulness of the simulation program.

Method of Approach

A Monte Carlo simulation of the system was developed to compare the various alternatives via the simulation output data. General Purpose Systems Simulator II (GPSS II) was the simulation language (17) and the program was run on the UNIVAC 1108 computer at the Rich Electronic Computer Center at Georgia Tech. Data used in the simulation were provided by the company, and included interarrival times, areas called, and call holding times. Both interarrival times and holding times were expected to be exponentially distributed in keeping with most literature on the subject. Conformity tests (see Appendix B) on the actual distribution of call data showed that it indeed followed the exponential density.

Simulation of the telecommunications system from call generation to call completion was used to provide output data on delay times, WATS line utilization, and occurrence of toll calls. For each run the maximum delay time, as defined by the system usage policy, and the line mix were varied so that each line mix was run with each maximum delay time possibility. A quasi-smoothing process was used in part to determine the successive line mix alternatives to be simulated since preceding output data gave clues as to which areas needed reinforcement. To check for random error, each mix was simulated for three days, which in effect caused the program to be run with a different random number seed three times. The simulation runs for the original mix of five Zone 4 and six Zone 6 lines were further validated by comparison with real data on average number of calls placed by zone per day.

The line mix alternatives were compared in terms of the important variables--fixed cost, toll call occurrences and cost, line utilization, average delay times, etc. The line mix yielding minimal cost, where total cost is the sum of the line lease costs and the expected toll call costs, while retaining desirable serviceability standards, was determined for both maximum delay time alternatives.

CHAPTER II

LITERATURE

Teletraffic Theory

The foundations of teletraffic theory and consequently queueing theory were laid by A. K. Erlang, a Danish mathematician, early in this century. At that time the technological development of the automatic telephone exchanges required the use of probability theory to determine the number of telephone lines between the central offices that would afford reasonable service to the customers (4, p.159). Erlang's first paper, which appeared in 1917, introduced methods by which he could obtain the probabilities of different numbers of calls waiting and of the waiting time, and the probability of lost calls in a system allowing no queue. These methods were developed under the assumptions of Poisson input from unlimited sources, constant or exponential holding times, full availability, and the system being in a state termed "statistical equilibrium" (12, p.6).

Erlang's initial work in the new field of telephony stimulated the interest of others who continued to develop the theory for more complicated situations of different arrival patterns, service mechanisms, and queue disciplines. A succinct history of the progress in telephony and queueing theory is presented in Saaty's book *Elements of Queueing Theory* (13, p.20-25). One of the more comprehensive treatments of this theory is contained in Syski's book, *Introduction to Congestion Theory* *in Telephone Systems* (15). With regard to the problem presented in this thesis, a number of concepts in teletraffic theory are important. Of particular interest is the continuing investigation of *arrangements in echelon* and *graded multiple systems* initiated by R. I. Wilkinson (1931) and C. Palm (1936). Both systems involve servers arranged in ordered groups wherein a demand is served by the lowest ordered group having a server available. The simplest system of this type is that of "divided access" where there is one first choice group and an infinite second group. This infinite second choice group provided the theoretical basis for Wilkinson's development of an approximation technique, known as the Equivalent Random Method, for the description of overflow traffic. This procedure "depends upon the fact that a given pair of values B_1 and V_1, representing an overflow, corresponds to a pair A and R representing the input and size of a single group of servers" (12, p.120). This is an improvement of the approximation where traffic overflowing from one part of a grading is regarded as being Poisson distributed with appropriate mean, and Erlangian theory is then used to calculate the loss (13, p. 311).

Studies on the conditions of overflow have been made by many, including Palm and Takács. Another area of study pertinent to this thesis problem is that of defection of delayed calls. Palm (1937) was among the first to consider defections. D. Y. Barrer (1957) considered the problem of impatient customers who leave the queue after a fixed wait, as in the MDT system, and derived the corresponding queue-length distribution (13, p.28).

The probabilities of state have been the object of most mathematical consideration. The system state may be defined, in the simplest cases, by the number of busy servers (in a loss system) or the number of customers waiting and being served. While these system states vary in time, primary attention for technical purposes has been paid to probability states in statistical equilibrium. Statistical equilibrium is "the condition in which the system's probabilistic description is invariant in time" (12, p.22). This steady state concept can be applied successfully to the busy hour in telephony--the period of greatest interest.

Of central importance to the probabilistic treatment of traffic sources and service is the relationship of the Poisson distribution and the negative exponential distribution function: the interval of time between random arrivals, as described by a Poisson distribution function, is exponentially distributed (7,12). Without the use of the exponential distribution either in describing interarrival times or duration of service times, few "nice" results have been found mathematically (12, p. 4). The simplest such traffic system involves an infinite number of servers in statistical equilibrium, as first treated by Erlang. However, for a finite number of servers consideration of the single server system is important in that it can serve to introduce ideas and methods useful for these more complicated systems.

At the heart of the mathematical consideration of the loss and delay systems are the two formulas developed by Erlang, the Erlang Loss Formula and Erlang Delay Formula. Consider a system where customer arrivals follow a Poisson Process with rate λ, service times are exponentially distributed with mean service time μ^{-1}, and customers who find none of the s servers available simply leave the system. In statistical equilibrium, the probability of j busy servers is

$$P_j = \frac{(\lambda/\mu)^j/j!}{\sum_{k=0}^{s} (\lambda/\mu)^k/k!} \quad (j=0,1,\ldots,s) \tag{1}$$

$$= 0 \quad (j>s)$$

where (λ/μ) is the offered load, often denoted by a. The offered load is a dimensionless quantity but is numerically expressed in units called *erlangs*.†

In the case of j = s, the formula becomes

$$B(s/a) = \frac{a^s/s!}{\sum_{k=0}^{s} (a^k/k!)} \tag{2}$$

--the Erlang Loss Formula. B(s,a) states the probability that all servers are occupied and, according to a well-known theorem, since customer arrivals are Poisson, this is also the probability that an arrival will find the system full and therefore be lost. Families of curves showing the probability of loss, and equivalently the proportion of time that

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†One erlang = 36 ccs; 1 ccs = 100 call-seconds.

all servers are busy, for various values of a plotted against fixed numbers of s servers are available [see Cooper (5)].

The carried load a' is defined by

$$a' = \sum_{j=1}^{s-1} jP_j + s \sum_{j=s}^{\infty} P_j. \tag{3}$$

It can be interpreted as the portion of the offered load that is not lost. In the case of a loss system, Equation (3) simplifies to

$$a' = a[1-B(s,a)]. \tag{4}$$

These results are discussed in Cooper (5).

For purposes of measurement, the quantity a', the carried load, has been quite useful. For example, measurement of the load carried by a group of telephone operators is accomplished by a device connected to the incoming trunks called a traffic usage recorder (TUR). The load carried by each operator is expressed by

$$\rho = \frac{a'}{s}. \tag{5}$$

ρ is also called the utilization factor.

Although the derivations of the above formulae for loss systems are usually made assuming an exponentially distributed service time, the results are valid for any service time distribution provided the input follows a Poisson Process. However, arbitrary service time distributions cannot be used when expressing Erlang delay probabilities (5, p.69).

In the Erlang delay systems, an infinite number of waiting positions is assumed, and, since no customers are lost, the offered load is equal to the carried load. Given a mean service time of μ^{-1}, the service completion rate is dependent upon the number of customers in the system:

$$\mu_j = \begin{cases} j\mu & (j=0,1,\ldots,s) \\ s\mu & (j=s,s+1,\ldots) \end{cases} \tag{6}$$

In a state of statistical equilibrium,† the probability of all s servers being busy

$$\sum_{j=s}^{\infty} P_j = \frac{a^s}{s!}\frac{s}{s-a}P_o \qquad (0 \le a < s) \tag{7}$$

where

$$P_o = \left[\sum_{k=0}^{s-1}\frac{a^k}{k!} + \frac{a^s}{(s-1)!(s-a)}\right]^{-1} \qquad (0 \le a < s) \tag{8}$$

is expressed by the Erlang delay formula, $C(s,a) = \sum_{j=s}^{\infty} P_j$:

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†If the offered load a, and in turn the carried load a', were greater than or equal to s, there would be an infinite queue and consequently no statistical equilibrium distribution (5, p. 72).

$$C(s,a) = \frac{a^s/[(s-1)!(s-a)]}{\sum_{k=0}^{s-1} (a^k/k!) + a^s/[(s-1)!(s-a)]} \quad (0 \le a < s) \tag{9}$$

It becomes evident in working with cases of practical interest that the state equations can often be specified with little problem, but are not easy to solve in closed form. Another tactic, that of indirect solution, sometimes provides means of solution. Cooper (5, p.122) has outlined a method of calculating the load $a'_{bcc}(m)$ carried by the mth server in a loss system by considering it as the "difference between the load aB(m-1,a) overflowing the (m-1)th server and the load aB(m,a) overflowing the mth server" (5, p.123), i.e.

$$a'_{bcc}(m) = a[B(m-1,a)-B(m,a)]. \tag{10}$$

B(m,a) is the Erlang Loss formula. Had this form of solution not been discovered, the direct method would have involved determining the difficult distribution P(i,j,k) describing the probability of servers being occupied.

Expanding the alternate solution to an Erlang Delay system involves separating the time intervals into two mutually exclusive and exhaustive sets: one when there are no customers waiting and the other when there is at least one customer waiting. Since the behavior of the system with customers waiting is independent of the behavior when there are no customers, it was shown that the load carried by the mth server in a delay system can be calculated from the formula

$$a'(m) = p(s+1,a) + [1-p(s+1,a)]a'_{bcc}(m) \quad (m=1,2,\ldots,s) \qquad (11)$$

where $p(s+1,a) = \sum_{j=s+1}^{\infty} P_j$, P_j being the equilibrium distribution of the number of customers in the system (5, p.124).

As reported by Cooper (5, p.147), P. J. Burke (1963, unpublished) originated this idea and used it in his study, "Economic Engineering of Flat-rate and Measured-rate Trunks." In his example, the first choice group was flat-rate (WATS) lines, and second choice was measured-rate (WATS) lines, and in the case that neither was available, the calls would queue with an operator until a line from one of the groups was available. The objective was to establish the optimal economic mix by comparing the costs of various mixes. To do so, Burke calculated the load carried on the c flat-rated trunks and the s measured-rate trunks, varying the values of c and s. The formula he derived to calculate the load on the s measured-rate lines was

$$L(s) = s\frac{a}{s+c}C(s+c,a) + a\left(1-\frac{a}{s+c}C(s+c,a)\right)[B(c,a)-B(s+c,a)]. \qquad (12)$$

Burke's problem is closely aligned with the one of this thesis; however, there are some major differences. Since the object of this thesis was to develop a method that would identify a good mix of lines for a particular company and involved a more complicated set of alternatives than Burke's, it was felt that an analytical solution based on Burke's formulation was not readily applicable. Further derivations for formulae would be required to account for the graded effect of the WATS zones, and for the maximum delay time requirement. Further, in order to approximate the states of statistical equilibrium required by such formulae, segments of a day of operation would need to be analyzed separately. Consequently, it was decided that the best solution of the problem could be made through simulation of the system.

Simulation Techniques

Literature specific to the problem of simulating a WATS system to determine the optimal mix of WATS lines is scarce. J. K. Klitz and A. W. Mecklenburg of IBM (8) did a simulation study for that company which they presented to the 1971 AIIE National Convention. Their goal was identical to that of this thesis: to develop a method of optimum WATS line selection for their company. "Optimum" here implies the correct number of WATS lines and combinations to provide specified acceptable service at minimum cost.

Klitz and Mecklenberg used both integer programming and simulation to first bound the possible combinations of lines, simulate usage, and then cost out the acceptable combinations within those bounds. Phillips (11) commended the study as a "sincere effort to apply sound operations research methodology to the solution of a real world problem." However, he went on to say that the integer programming formulation used to restrict the possible combinations of lines could have been supplanted by simple Gauss-Jordan elimination procedures, and consequently was not a significant analytic contribution to the problem.

The cost function involved a fixed service (leasing) cost plus a Delay Cost. Delay cost was computed as:

$$\text{Delay Cost} = (\text{Probability of Toll Call}) \times (\text{Callers Waiting}) \times (\text{Cost of Toll Call})$$

The authors' premise was that the cost of a toll call represents the greatest expense that can be incurred due to delay.

Further, Klitz and Mecklenberg state that the probability of using toll facilities because of delay is a management policy. For purposes of simulation, Klitz and Mecklenberg interpreted this management policy as follows (8, p.268):

1. All users would be expected to wait at least .5 hours.
2. 20 Per cent of those waiting between .5 hours and .75 hours would be critical and necessitate a toll call.
3. 60 Per cent of those waiting between .75 and 1.0 hours may be expected to place a toll call.
4. 80 Per cent of those between 1.0 and 1.5.
5. Everyone waiting more than 1.5 hours should use the toll system.

Monte Carlo Simulation was used in computing delay time.

The authors then stated the algorithm used in their study:

1. Compute peak arrival rates + margin; mean arrival rates.
2. Solve the integer LP model for upper and lower bound L (yields service cost).
3. Simulate the delay, . . .
4. Compute the cost of delay . . .

5. Compute L which yields the smallest reduction in service cost from the previous one.

6. Go to 3 until the incremental increase in delay costs begins to far exceed the incremental reduction in service costs. When this occurs a solution has been found; or until the lower bound service cost has been reached--select minimum.

GPSS III was initially used for the simulation. Later, the finalized version was done in GASP II, a Fortran based simulation language. The authors reported that it ran 10-12 times faster than the GPSS III (11).

Klitz and Mecklenberg emphasize the value of such system analysis. They found the "intuitive methods of call assignment and band assignment are usually non-optimal and sometimes poor." Also, after analysis, they recommended that utilization of any WATS band should not exceed 80 per cent; otherwise the service rapidly deteriorates.

Finally, Klitz and Mecklenberg speculated that a toll call spill-over during peak hours might prove economically more reasonable than providing WATS lines to accommodate peak hour traffic.

Subsequent conversion[†] with Mr. Klitz revealed that this study resulted in a reduction of WATS lines from the eight recommended by the Telephone Company to five, while retaining an acceptable level of service.

Sinclair Oil Corporation made use of GPSS Simulation in modeling their private line communications network. This is not a WATS system, but it does closely parallel such a system. Marvin Ames (1) reported

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[†]Personal Telephone Conversation, November, 1971.

that Sinclair's system is directly accessed by the caller and no queues are formed. If a required line is "busy," the caller just keeps dialing until access is made. Delay times are established from the amount of time between retries. Interarrival time is assumed exponentially distributed as is holding time on a successful call.

Data on calls were gathered via an annual survey of the some 2500 users of the intracompany system. This data became a part of the Survey Analysis Support Program which, combined with a Path Generation Program and Transaction Generation Program, provided an external source of calls for the GPSS Simulation Program.

Ames detailed the methods of validation, primarily comparison to real data, and then reported that a 7 per cent improvement was projected for system performance if the recommendations resulting from the simulation study were implemented.

James B. Thies (16) used a WATS System Simulation to illustrate the potential of computer modeling and simulation as a management tool. This particular simulation represents only a small part ("micro element") of a Simscript model of Field Enterprises Education Corporation (World Book, etc.). Just as was done by Klitz and Mecklenberg for IBM, this model was built "entirely from scratch."[†]

Thies defined the system and the potential alternatives to be tested. The basic objective was "to minimize the cost of long distance telephone communications by balancing the cost of (1) WATS lines, (2) toll calls, and (3) waiting time of telephone users. Thies related the

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[†]Personal written communication, December 1, 1971, J. B. Thies.

cost of waiting time--delay time--to opportunity cost. This is contrary to the premise of Klitz and Mecklenberg that delay time is most expensive in terms of what a toll call would cost. Thies' formulation of the WATS optimum mix problem more closely follows the objectives of the company under study herein than does the Klitz-Mecklenberg model because of the waiting time/opportunity cost link. Also, Thies used waiting time as a variable in his simulation, while Klitz and Mecklenberg did not vary their waiting time criteria from their five-stage company policy interpretation.

Once the simulation was run, the reported validation of the model was extensive and thorough. Much emphasis was placed, of course, on comparison of output data with real-life data.

A number of approaches to the technique of presenting data to management were outlined. These included:

1. Applying a cost function to the alternatives.
2. Using a Response Surface to show sensitivity of dollar costs to decision variables.
3. Plotting time-path data to show (a) probability of success in obtaining a WATS line, and (b) percentage idle time of WATS lines.

The main thrust of Thies' article concerns the value of smaller system designs and analyses in breaking ground for more extensive modeling and analysis for use in aiding management decision making. Communication with Thies revealed that the technical aspects of his modeling procedure were deleted in the article which appeared in *Financial Executive*. Consequently, the power of the published report lay in illustrating methods of validating and presenting technically based analysis to management in terms they themselves can relate to the situation. Selling the decision-makers on the results of such a technical analysis is as important as modeling the problem--even today it is difficult to convince management that results contrary to their intuition can be and often are correct.

Karl I. Nordling (9) provides useful information on the various Long Distance Telecommunications services available, as well as their relative costs. He reiterates that optimum service can best be attained through system analysis. Nordling's contribution to this type of analysis is a comparison of Direct Dial (Toll), measured and full-rate WATS, and Private Line service (which is what Sinclair Oil used). He includes charts on rate structures that could be used to compare the relative costs of these alternatives. On WATS versus Toll, Nordling concludes that WATS is less costly if a number of call requests arriving together can be placed sequentially, as from a queue, rather than simultaneously, and if the monthly usage exceeds 50 to 70 hours.

CHAPTER III

THE MODEL

The Real World

The company under study now operates with a priority system of WATS line usage. The information for the call, i.e. name of requestor, his number, area and number to be called, is taken by an operator and recorded on a call ticket. If a line is available and no call is pending, she can immediately place the call. If a line for that zone is not available, the call request is placed, in order of priority, with the other tickets waiting for those lines, and the operator informs the requestor that she will call him back when she can access a line.

Priority of the calls is defined in the "Switchboard Standard Operations Instructions" prepared by the telecommunications staff as follows:

1. Emergency (fire, life or death).
2. Priority Business (Corporate), Company President, other high ranking officials.
3. Priority Business (RI number assigned, or project number for which prior approval in writing has been given).
4. Routine Calls (to be maintained and placed in order of time the call was placed).
5. Personal Calls (not to be placed over the WATS Lines during normal duty hours unless instructed to do so).

In addition, these instructions state that "Direct Distance Dialing (DDD) should not be practiced except in extreme workloads during peak traffic hours."

When a line becomes available, the operator takes the next ticket in sequence and tries to contact the calling party. Upon reaching the requestor, the call is immediately dialed. It has been estimated by a company representative that 10 per cent of the time the operator cannot locate the caller. If his extension is not answered or is busy, the operator is instructed to hold his ticket one call and try again. The number of times she tries unsuccessfully to contact the calling party is "strictly dependent on the priority of the call (mostly Operator judgement)."[†] If a secretary answers and the caller is not available, the operator leaves a message for him to call back to place the call. In these cases the ticket is put in a "hold status."

All the operators are expected to see that the WATS lines are utilized to the utmost. This results in what is termed an "adaptive priority decision process"[†] in assigning WATS lines to call requests. The lowest rated line that will carry the call is normally the choice, but if it is apparent that the waiting time for one area is becoming excessively more than for a higher rated or "more expensive" zone, the tickets are spread to the queues before higher rated zones to help equalize delay time on the calls. At this time the company has lines from only two zones available, Zone 4 and Zone 6.

Other types of long distance calls placed from this company are Special Billing calls (bill to a third number, credit card, etc.) and Person-to-Person WATS calls. During a week's survey of the company

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[†]"Switchboard Standard Operations Instructions."

calls, there were reported 16 Special Billing and 12 Person-to-Person WATS calls. Such a small number of calls would in no way affects WATS usage or the number of lines needed.

A more important aspect of WATS line utilization is the number of persons who prefer not to wait for a WATS line and request that the call be placed on Toll lines. In the "Instructions," such requests, which are discouraged, must be referred to the Chief Operator with an Operator with an explanation of the urgency. Other factors, such as the time of day and the number of times the call was placed in a "hold status" due to the caller not being available, are supposed to be considered in releasing a toll call. In practice, however, a company representative reported that "Generally, if a person had to wait more than ten minutes he would then take the toll call. There were a number of persons who would not wait at all. This was about 5 per cent and there were some who waited up to four hours before going toll."

A recent study of one month's toll call bill revealed that the total "unfixed" cost of telephone long distance service or WATS system spill-over was about $3,600 for approximately 1000 calls. This averages to some 50 calls per day spill-over.

In addition to the operator link in WATS line use, there exists a direct access feature whereby persons connected with important projects, or those in higher management can dial a code and obtain a WATS line if one is free. If a busy signal is reached upon dialing the code several times, the calling party will usually place his request with the operator. There are differing opinions as to the value of a direct access feature during business hours. It is convenient for the caller, but it eludes systematic regulation of WATS usage. Also, it becomes inconvenient for the operator who, when ready to dial a number, loses her line to someone using a direct access code. The effect of direct access will be tested on several line mixes with the simulation program.

The Simulation

The transactions in the simulation are call requests. Generation of a transaction represents the caller placing his request with the operator. The interarrival times between these call requests have been produced for the simulation using an exponential density having its parameters estimated from actual data compiled by the company for this study. Interarrival distribution parameters vary with both the area being called and the time of day.

Unless a line from the designated area is immediately available, the call request is placed in queue before its first choice group on a first-in-first-out basis. The call request will not leave the queue until (1) a line of the first choice group is available and the call is completed, or (2) the maximum delay time is reached. Several routes for completing the call after the MDT has been reached are considered. If the time of day is before 11:00 a.m., the transaction tries to use a WATS Zone 6 line. Since Zone 6 covers the West Coast which "opens up" three hours later than the East Coast due to the time difference, traffic for that area is not generated until 11:00 a.m. EST. If all Zone 6 lines are in use, or if it is past 11:00 a.m., the transactions search for the first available line in a higher rated zone. If that search is not successful, the call is immediately completed over a toll line.

Completion of a call is defined as the match between a caller and an available line, i.e. as soon as the call is dialed on a line, the call request can be considered terminated. The holding time for the call simply regulates line availability. In cases where a re-try is necessary due to busy signals, the re-try will be considered a newly generated call request. Since no data were available on how often the number dialed was busy, or whether or not the interarrival times data included re-tries, it was felt that a guess as to the probability of this occurrence with resultant recycling, would not enhance the program's validity.

The probability of the operator being unable to contact the requestor when a line is free for his call was included in the simulation. Under the present system it was estimated that this would occur 10 per cent of the time.[†] The company concurred, however, that if the caller would expect a connection within a specified delay time, the chances of his leaving or otherwise being unavailable during that time would be considerably lower. Accordingly, the percentage of unavailability has been adjusted to 4 per cent prior to reaching maximum delay time (MDT) and 6 per cent when MDT has been reached. Such calls will be assigned a higher priority and recycled.

Each simulation day runs from 8:00 a.m. until the calls in progress at 5:00 p.m. complete their holding time. Any call requests in queue at 5:00 are erased. The time incrementation used was one-half

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[†]Best estimate of a company representative.

minute (30 seconds).

Description of Flow

Six pseudo-generators are used to generate the call requests, one for each WATS zone. Zone 1 calls for example are created by block 1 (see Figure 2). Facility 11 is the dummy facility of the pseudo-generator, and the interarrival time is computed in block 21. Block 31 assigns the natural WATS zone to Parameter 2 of the transactions. This is later used to determine the zones into which toll calls are made. Block 41 assigns the first choice zone number to Parameter 5. Parameter 5 will vary with the line mix, but for the original mix of five Zone 4 and six Zone 6 lines, Zone 1 calls go on a Zone 4 line, so P5 = 4. Also for Zone 2, 3, and 4, P5 = 4. For Zones 5 and 6, P5 = 6.

Blocks 81 and 82 in combination with blocks 92-98 provide the means to test the direct access feature. Blocks 81 and 82 assign priorities, retained in P3, on all call requests. It is assumed that 60 per cent of the calls of Priority 12 or greater try to directly access WATS lines. Of those who dial the direct access code, some will find all the lines for that area (P5) busy and will then place the call with the operator in the normal way, via Block 140.

All transactions enter Storage 10 through Block 91. This storage must be empty before succeeding days of simulation can commence. All transactions leave Storage 10 through Block 450 before terminating.

Block 140 adds all entering transactions to a SAVEX cell (a storage) determined by Parameter 2, the natural zone number. When a call is completed, the number of calls recorded in the appropriate savex cell is reduced by 1. These SAVEX cells are sampled every 15 minutes by a transaction created in Block 360 (see Appendix A). These samples are useful in determining which natural zone queues are building up and might need more outgoing WATS lines.

Block 400 holds the call requests in the queue specified by P5. These calls can leave their queue by one of the following routes:

1. The time of day is 5:00 p.m. and all remaining call requests must leave the main storage 10 and be terminated (Block 149).
2. A line in the desired Zone becomes available and the operator tries to connect the caller (Block 150).
3. The waiting time in the queue reaches the maximum delay time and the call request is routed to either a higher rated line or a toll line (Block 151).

If, in the second case above, a line is obtained there still remains a 4 per cent chance that the caller is not available. Blocks 157 and 158 increase the priority on such a call request, count the number of times it has been that same route, and recycle the call. In the event that that transaction has been through four times, it is automatically terminated. The other 96 per cent of the call requests that obtain a line are advanced, hold the line for a time computed in Block 165, are tabulated in Block 301, and terminated in Block 98.

The call requests that reach the maximum delay time advance to Block 220. Before 11:00 a.m. the calls can go on a Zone 6 line if available. Otherwise the search for an available higher-zone line begins in Block 229 and an indexing arrangement is used to check each zone. When all higher zone lines have been tried, the call is placed over Toll lines, Blocks 242, 269, 270. The toll calls are tabulated in

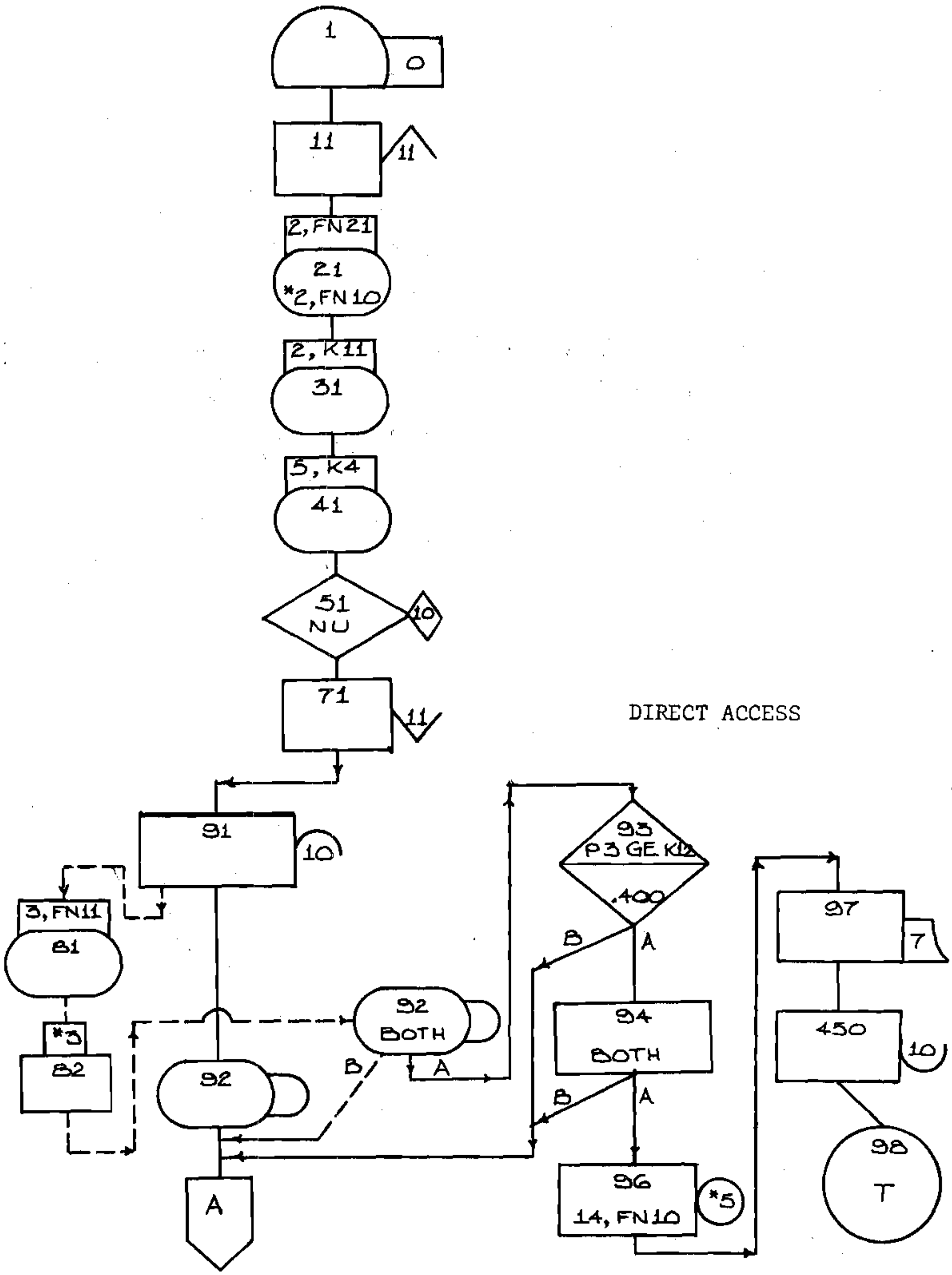

Figure 2. Flow Chart of WATS Line Mix Simulation Program

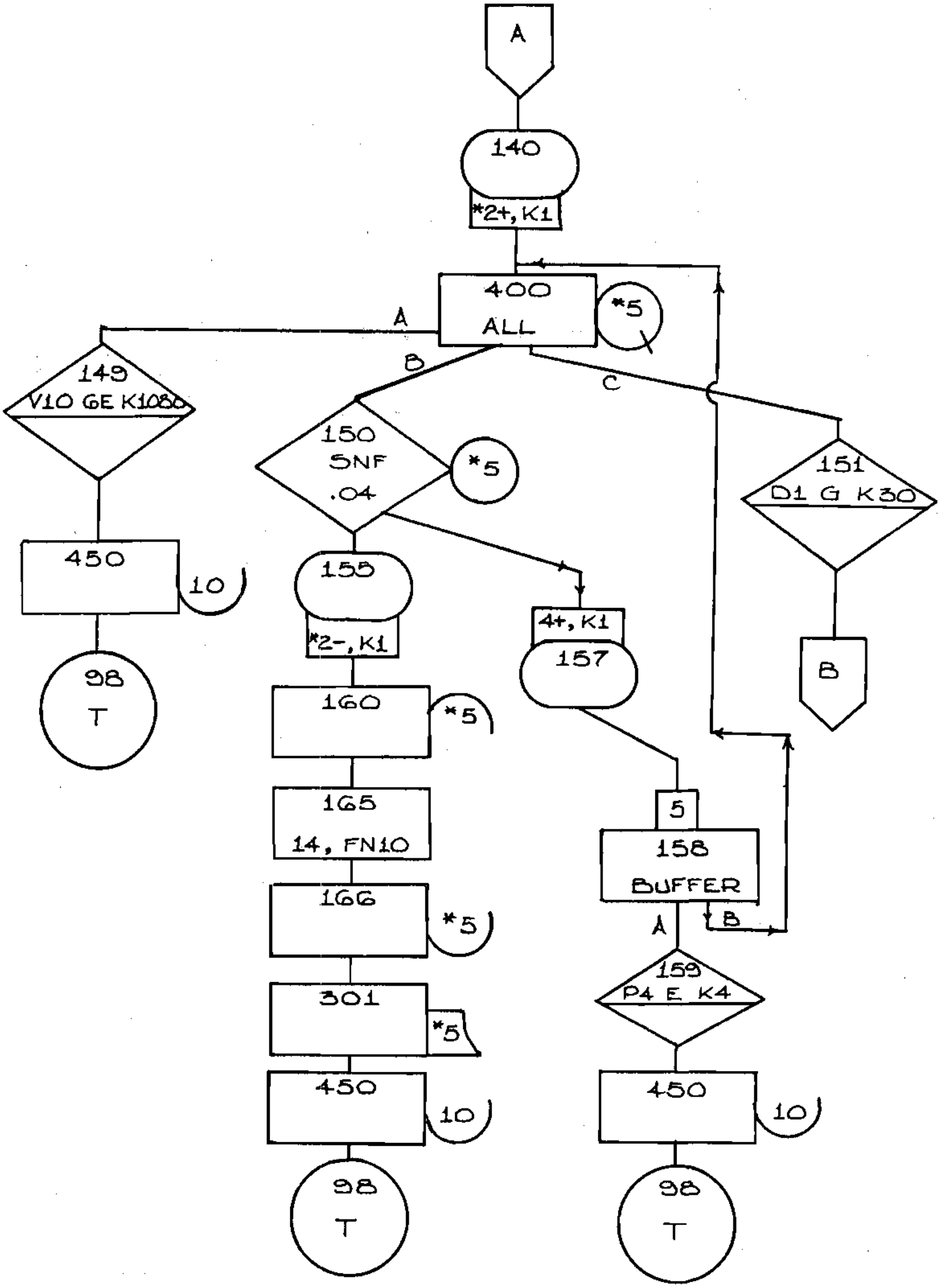

Figure 3. Flow Chart of WATS Line Mix Simulation Program

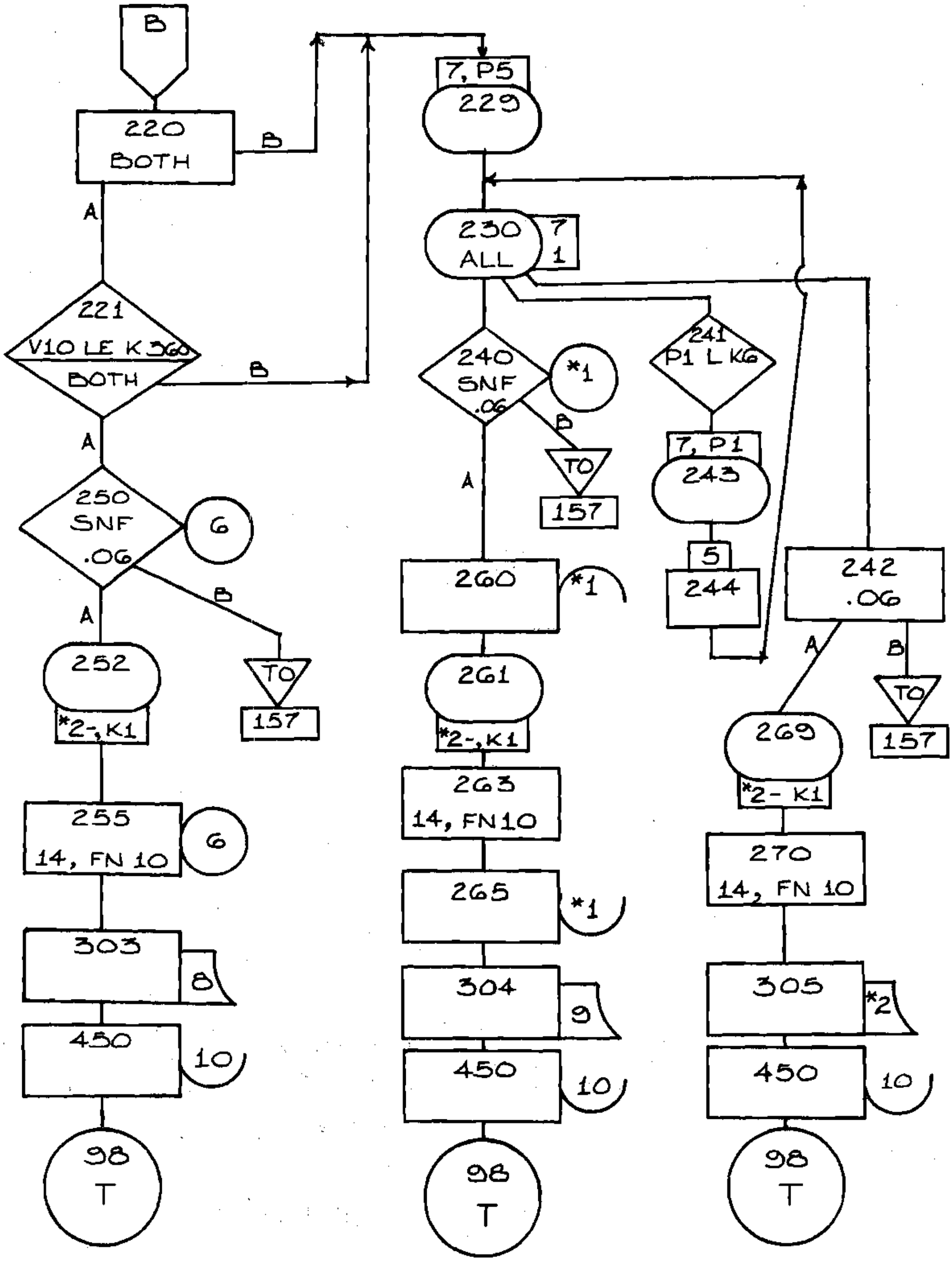

Figure 4. Flow Chart of WATS Line Mix Simulation Program

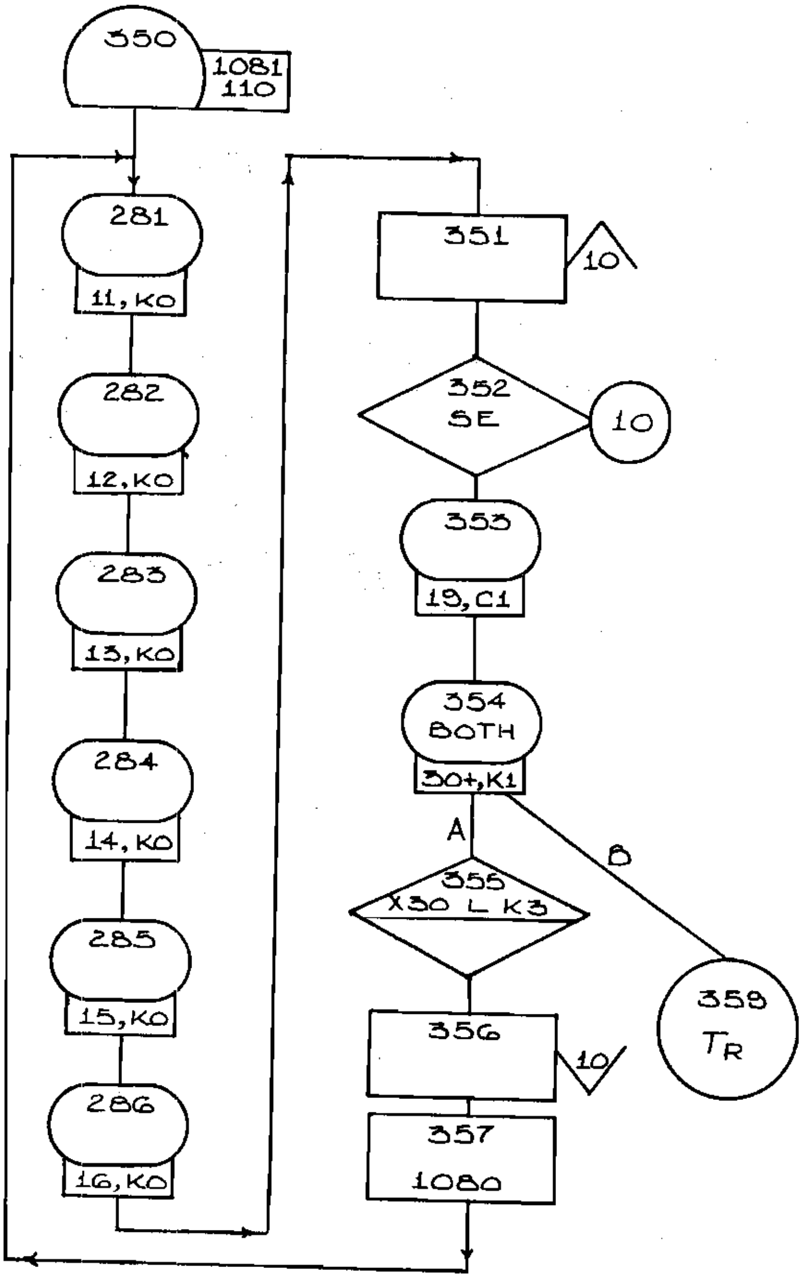

Figure 5. Flow Chart of WATS Line Mix Simulation Program

Block 305 according to P2, the natural zone number.

Blocks 350-359 close the board at 5:00 p.m. by shutting the gates in Blocks 51-56. Once Storage 10 has been vacated, all SAVEX cells 11-16 are set to zero, the time of day as contained in Variable 10 is adjusted back to 0 (8:00 a.m.), and the board is reopened. Blocks 354 and 355 terminate the simulation after the desired number of days (K3 = 3 days).

Blocks 371-376, 381-386 (see Appendix A), define the storage for Parameter 5. To indicate that there are no lines for a particular zone, e.g. Zone 5, a capacity of 1 is assigned to Storage 5, and a single transaction is generated in Block 375 which is held throughout the entire simulation run in Block 385, Storage 5. If there are lines in a zone, the transaction created will have a zero action time, as in Block 384.

Statistics

In the discussion of "Data Input and Simulator Validation" Schmidt and Taylor (14, p.479) recognize three categories of variables: decision variables, primary random variables, and secondary random variables. Decision variables are defined as those whose "values are completely controlled by personnel within the system." For example, the decision variables in this case are the various mixes of WATS lines being tested, and the 2-3 different maximum delay time alternatives being used. The secondary random variables are those whose values are completely related to the values of the other random variables. In this simulation, the secondary random variables provide the information on which to base a WATS mix recommendation: the number of toll calls generated, the average time in queue, percentage utilization of the lines, average delay time, etc. These all are values which have been calculated within the program. Primary random variables, as opposed to secondary random variables, are generated rather than calculated. The rate of incoming call requests and the service times on completed requests are the primary random variables in this study, and their distributional properties are to be determined from sample data.

Call arrivals in Erlang loss and delay systems are assumed to be Poisson distributed; in fact, without this assumption few "nice" results have been found mathematically. The probability distribution function of the Poisson distribution, or the probability of x calls occurring in time t is:

$$f(x) = \frac{(\lambda t)^x \, e^{-(\lambda t)}}{x!} \, ; \qquad \begin{array}{l} \lambda > 0 \\ x = 0, 1, \ldots \end{array} \tag{13}$$

$$= 0 \qquad \text{otherwise}$$

where λ is the mean arrival rate. Properties of the Poisson distribution include the fact that the mean and variance are both λt.

As mentioned earlier, there exists an important relationship between the Poisson distribution and the negative exponential distribution: the interarrival time between arrivals following a Poisson process is exponentially distributed. This well-known relationship is derived by Cox and Smith (7, p.9-10) in a particularly instructive manner. The probability density function of the exponential distribution with mean $\theta = 1/\lambda$, where λ is the mean arrival rate of the corresponding Poisson distribution, is:

$$g(x) = \frac{1}{\theta} e^{-x/\theta} \qquad \begin{matrix}\theta>0\\ x>0\end{matrix} \qquad (14)$$

$$= 0 \qquad \text{otherwise.}$$

The mean θ and variance for this distribution are also directly related: the variance is the square of the mean, or θ^2. The service time distribution of the duration of telephone calls can also be approximated by the exponential distribution, particularly if there are a larger number of customers requiring relatively short service times and a smaller number of customers whose service time is longer (7, p.20). However, departures from the exponential can be expected from such causes as the charges per 3 minute interval for toll traffic (3, p.42).

The rate of incoming call requests for the different WATS zones and the duration of successful calls used in the simulation program were developed from data collected by the company during June, 1971. For five days of that month (1 Tuesday, 4 Wednesdays) the switchboard operators kept a record of the number of minutes between call requests for each "natural" area and the holding time on each "completed" call. These data were subsequently assembled into frequency distributions to help identify the functional distributions. The frequency distribution for holding times, for example, is shown in Figure 6. It appears to follow a negative exponential distribution. Further indication of its exponential distribution comes from calculating the mean and variance of

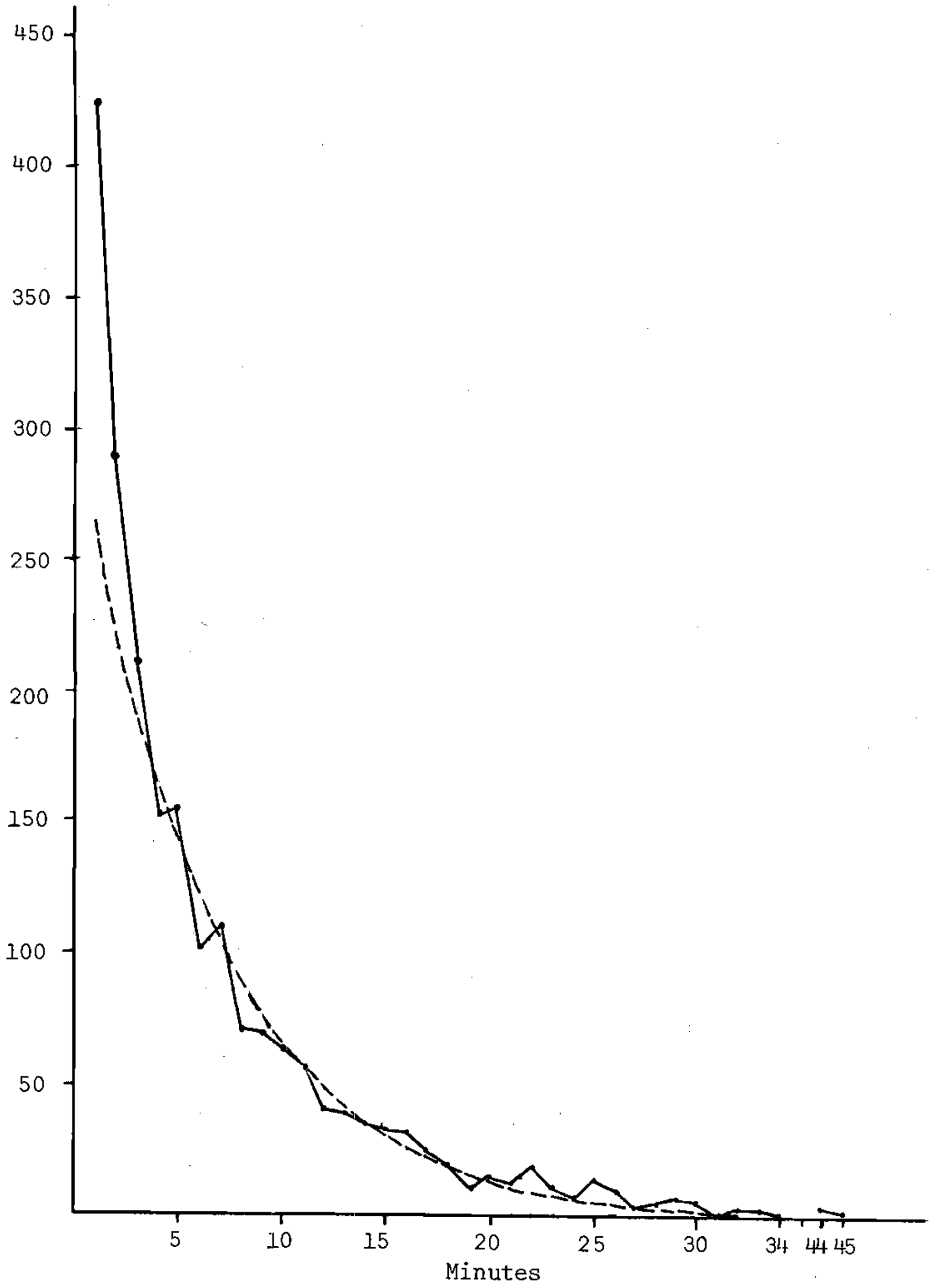

Figure 6. Frequency Distribution of Service Times Data ($\bar{x}$=6.47)

the sample data. For the holding time data, the sample mean was calculated to be 6.47 minutes. The sample variance was 41.36; the square of the sample mean is 41.86. Thus the relationship between the mean and variance of the exponential distribution appears to be satisfied for the holding time data (see Appendix C for calculations).

To show that the data compiled for this study do indeed conform to the exponential distribution, the Chi-Square Goodness-of-Fit test was applied to (1) the interarrival times by area and (2) the holding times. In testing for the conformity of the interarrival times to the exponential distribution, the data for each zone was compiled and tested on a whole day basis rather than by hours of the day. This was necessary because there were at times not enough data for each hour to accurately test its distribution. The results of the Chi-Square test are summarized in Appendix B. The interarrival times by zone all tested out to the .975 percentage probability in conforming to the exponential distribution. Consequently, the interarrival times were generated exponentially by including the modifying exponential function in the simulation program.

Upon inspection, the interarrival times exhibit a non-stationary mean during the day. Accordingly, these changing means have been accommodated in the simulation program through the use of Functions 21-26 (see Appendix A). Table 1 shows the various interarrival time means for the hours of the day.

Table 1. Interarrival Time Means, θ

<table>
<tr><th></th><th>8 am</th><th>9 am</th><th>10 am</th><th>11 am</th><th>Noon</th><th>1 pm</th><th>2 pm</th><th>3 pm</th><th>4 pm</th></tr>
<tr><td>1</td><td>7</td><td colspan="3">6</td><td colspan="3">8</td><td>7</td><td>8</td></tr>
<tr><td>2</td><td>4.5</td><td>3.5</td><td>5</td><td colspan="2">6</td><td>4</td><td colspan="3">5</td></tr>
<tr><td>3</td><td>4.5</td><td colspan="3">3.5</td><td colspan="2">6.5</td><td>5.5</td><td>4.5</td><td>4</td></tr>
<tr><td>4</td><td colspan="3">2.5</td><td colspan="2">3.5</td><td>2.5</td><td>3.5</td><td>2.5</td><td>3.5</td></tr>
<tr><td>5</td><td colspan="2">∞</td><td colspan="2">8</td><td>7</td><td>10.5</td><td>7</td><td colspan="2">11</td></tr>
<tr><td>6</td><td colspan="3">∞</td><td colspan="2">3</td><td colspan="2">3.5</td><td>3</td><td>3.5</td></tr>
</table>

The holding times data for completed calls in all zones were compiled on a whole day basis (8:00 a.m.-5:00 p.m.). The 2060 calls sampled yielded a mean of 6.47 minutes, and the data generally followed the exponential distribution, as indicated by Figure 6. Deviations from the exponential that were significant came at the holding times of one and two minutes. This can be explained by the fact that there are a large number of calls that are cut short because the called party is not available. Such shortened calls do not accurately represent true successful conversation time. No zero-length calls were recorded by the operators. The holding times after two minutes do follow the exponential distribution (see Figure 6), however, and no conflict is seen in using this distribution to describe call holding times in the simulator since calls cut short due to the unavailability of the called party are handled otherwise. The results of the Chi Square Goodness-of-Fit test are summarized in Appendix B.

In the simulation program, the mean time has been increased from 6.47 minutes to 7.0 minutes primarily to account for the down-time experienced between call connections. A sampling of calls made by the company revealed that an average of 20 seconds was used by the operator to obtain a line and dial the desired number. Since the unit of time in the simulation is a half-minute, 7.0 minutes seemed the best choice for this stationary mean. The down-time between calls and inclusively the "unavailable" time for the lines is accounted for not only by increasing the sample mean of 6.47 to 7.0 minutes, but also by the fact that the preponderance of short calls (1-2 minutes) in the real situation is not separately handled in the simulation.[†] It must be remembered that in this simulation, the holding time on calls provides the means of regulating line availability and as such, the importance of the holding times lies in the need to make WATS lines unavailable for certain periods of time to the call requests waiting to be served. Thus, the "service" time, initially approximated by actual call durations, becomes more than simply the representation of call holding times since it includes down-time between call connections.

Although the mean holding time of the sample does vary somewhat during the day for each zone, this variation should not be taken at face value. Since the number of calls in the sample during each hour of the day for each zone varies from 2 calls to 134 calls, the data for a

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[†]"Unsuccessful" calls due to the called party not being reached are still considered completed. However, re-tries on such calls are assumed to be newly generated call requests as in the case of busy signal calls.

particular hour can be distorted significantly by even one rather long or short call. Thus it was felt that the stationary mean time of the entire sample of 2060 calls would be most realistic.

Along with the three categories of systems variables used in a simulator there also are defined systems parameters. These are quantities which affect "the performance of the system, can be predicted with certainty, but either cannot or will not be altered by those operating the system" (14, p.482). The systems parameters of this WATS line mix problem are the fixed cost of leasing the unlimited usage WATS lines, and the unfixed cost associated with the toll call spill-over from the WATS system. The monthly cost for each interstate WATS line by zone is shown in Table 2. These rates are listed under FCC Tariff 259 and were effective February 1, 1970. The 10 per cent federal excise tax is included in the figures. While the monthly lease cost per WATS line is

Table 2. Lease Costs for WATS Lines from Florida (Full Service)†

Zone	Monthly Lease Cost††
1	$1,155.00
2	1,485.00
3	1,595.00
4	1,705.00
5	1,810.00
6	2,090.00

†FCC Tariff 259, February 1, 1970.

††Includes 10 per cent Federal Excise Tax.

constant for each zone, the cost of a toll call, varying with both its distance from the source and its duration, must be estimated.

Determining the approximate costs for the toll call spill-over involved associating long distance toll charges with the WATS zones for which toll calls were designated. Long distance toll charges are figured on the basis of rate steps. For any given area code and nnx (the first three digits of a telephone number, often called the "exchange") for both the calling party and the called party, there is defined a rate step. The rate step then designates the proper charge for the first three minutes[†] of conversation and the charge for each minute of conversation thereafter. To determine the rate step most closely corresponding to the WATS zones, a large number (a four-day sampling) of the call tickets written up by the switchboard operators for each call were reviewed to form a list of the area codes called by company employees. After obtaining the rate step for each area code, the area codes were separated according to the WATS zone they came within. This in turn showed the relative division of the zones by rate step. Table 3 shows the relationship between WATS zones, rate steps, and the cost of average toll calls. It also includes the average per-minute cost of WATS lines and cost of an average WATS call.

In computing the toll call cost, the toll calls were assumed to be an average of eight minutes in duration. Although the holding time on toll calls would follow an exponential distribution the same as WATS

[†]During the hours of 8 a.m.-5 p.m., the basic charge for DDD is for three minutes, while between 5 p.m. and 8 a.m. the basic initial unit of charge is one minute.

Table 3. Long Distance Toll and WATS Charges

TOLL CHARGES

Rate Step	Zone	First Three Minutes*	Each Add'l Minute	Cost for Average Eight-Minute Toll Call	+ 10% Tax
17	1	$1.00	$.30	$2.50	$2.75
18	2	1.05	.35	2.80	3.08
19	3,4,5	1.15	.35	2.90	3.19
20	5	1.25	.40	3.25	3.58
21	6	1.35	.45	3.60	3.96

WATS LINE CHARGES

Zone	Average Cost per Minute**	Cost for Eight-Minute Call Including Tax
1	$.12	$ .96
2	.16	1.28
3	.18	1.44
4	.19	1.52
5	.19	1.52
6	.24	1.92

*Rates Courtesy of Southern Bell, effective January 22, 1971.

**Cost figured from 80% usage of WATS lines, 9-hour day, 21 business days per month. Zone 6 lines more rightly should be figured on a 5-hour day which would place the per-minute cost at $.35.

calls, it was felt that an average holding time applied to all the calls would provide an adequate indicator of expected total cost of the toll call spill-over. Eight minutes was used as a mean rather than seven to help insure that the toll call cost would not be understated.

Validation

The simulator used in this study imposed hypothetical conditions on the system of WATS usage, primarily the maximum delay time criteria for using toll lines. As a result, the computer output cannot be tested against real informational data on the number of toll calls placed, the average waiting time in the system, etc. Areas which can be compared for program validity include the number of call requests generated per day (or several days), and the percentage of total call requests designated for each zone.

For all the runs for which data are available, the average number of calls per day was obtained by simply dividing the three-day total by three. In all cases (21 runs) the daily average fell in the mid-to-upper 700 range, the lowest being 734.67 calls generated, and the highest two being 794.33 and 777.67. Considering only the runs employing the 15 and 20 minute maximum delay times, the lowest average number of calls generated was 741.32; the high remained the same. The differences result of course from random variation caused by changes in the program statements, specifically the different MDT's and different line mixes. The average of all the 15 and 20 minute MDT runs (14 total) was 763.2 calls per day. Deviation of the high and low values from this average was approximately ± 3 per cent, indicating that the effect of random variation was not significant.

Actual data from the company reveal that on the average 732 call requests were made each day, of which approximately 22 per cent were retries.† Since this simulator considers retries as newly-generated call requests, these figures are consistent with the simulation results.

The percentage of total calls that were generated for each natural zone remained relatively constant by zone throughout all the computer runs also. On the average, the breakdown by zone is shown in Table 4. Also tabulated are the percentages computed from the interarrival time and holding time data. The most pronounced differences occurred in generation of calls to zones 1 and 3. For Zone 1, this may have been caused by the paucity of sample data for that zone, and possibly a greater deviation from the exponential. For Zone 3, some of the interarrival time means may have been overstated.

Table 4. Percentages of Call Volume to Each Zone

Zone	Program %	A Interarrival Time Averages	B Service Time Averages	Combined Average, A & B
1	11%	6.5%	7.1%	6.9%
2	17	17.5	17.1	17.3
3	18	22.8	23.0	22.9
4	29	30.5	32.2	31.1
5	7	7.1	5.1	6.1
6	18	15.6	15.7	15.7

†"Retries" were not defined by the company.

In addition to providing a means to compare percentages of calls to each zone against real data, the constancy of the simulated percentages provides further evidence that random variation in the program was not significant. Other consistences in secondary random variables through all the runs, such as percentage utilization, number of toll calls generated, indicated that random variation would be insignificant in relation to the decision processes.

Additional validation of this simulator was accomplished through agreement with company representatives on choices of alternatives and percentages applied to certain decision variables.

CHAPTER IV

EXPERIMENTATION

Experimental Design

This Monte Carlo simulation was designed primarily to produce information on the number of toll calls that would be generated by various mixes for inclusion in the cost model. However, since the cost of a system does not always identify the most desirable system, the program was designed also to provide information on delay times, on total time spent in the system, utilization of line facilities, and so forth. By comparing these variables of delay, average times in the system, percentage utilization of lines in a zone, percentages of call requests that met no delay in obtaining lines, etc., the various line mixes can be analyzed in terms of efficiency and usage patterns. Such analysis will insure that any line mix recommendation will not only satisfy the requirement of a more economical WATS system, but will also meet the demand for continued good telecommunications service to the users.

The various line mixes simulated have been represented in this text by "vectors" of six dimensions. For example, the line mix A which contains five Zone 4 lines, six Zone 6 lines, and no lines in Zones 1, 2, 3, or 5, has a vector representation of (000506). Table 5 lists the line mixes investigated in this simulation.

Table 5. Designation of LIne Mixes

Designation	Vector
A	(000506)
B	(002504)
C	(003404)
D	(022304)
E	(023403)
F	(022303)
G	(022403)

Some of the line mixes were arbitrarily selected, while others were chosen on the basis of information provided by the initial runs. Such information included which zones were showing the longest delay times and consequently more toll calls, what the line utilization factors were, and how balanced the queues were.

The seven line mixes were run at three levels of maximum delay time: 15, 20, and 30 minutes. The 15 and 20 minute MDT runs only will be presented here since the 30 minute MDT runs were made for information more than for purpose of recommendation. The 30 minute MDT run does not allow for shifting some call requests to higher rated lines before the MDT is reached. Arbitrarily waiting 30 minutes before spreading any tickets to higher rated lines which are not being used very much is not considered a desirable feature by the company, whereas a 15 or 20 minute delay can be tolerated. Of the 21 runs made, then, only 14 will be analyzed herein.

The program was written to simulate one day (8:00 a.m. to 5:00

p.m.) of operation. The time unit of simulation was chosen to be a half minute.[†] Using gates to "close the board," as described in the "Description of Flow," transactions (call requests) remaining in the system were terminated at 5:00 p.m. (or after 1080 time units of simulation) each day if they had not yet obtained a line. Those calls already on the lines were allowed to finish naturally. Once the system was emptied, statistics that indicated the time of day (Variable 10) or that were to be cumulated separately each day were set back to zero. In this way, each "day" of simulation started as a new day. The number of days simulated can be varied quite easily. Since the amount of computer time required to process the program went up significantly with each day simulated, it was decided to limit the simulation to three days. The simulation was not intended to reach a steady state condition as this would represent only the peak hours instead of the total day conditions. Consequently, the reiterations of the simulation were designed to smooth out random variation as well as to detect problems of significant random variation rather than to produce a steady state condition.

The Cost Model

The cost equations used for this WATS line mix problem are composed of the fixed costs of leasing the particular mix of lines plus the

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[†] Initially the program was run using one second as the time unit. For economy of operation, the time unit was changed to one-half minute (30 seconds). Since holding time and interarrival time data were recorded in terms of minutes, accuracy was in no way sacrificed by increasing the time unit to a half minute.

variable costs of toll call charges. Since the program simulates three days of operation, and an average month yields 21 working days (Monday-Friday), the total number of toll calls generated in each zone was multiplied by 7 in order to project the total number of calls per zone per month. On a monthly basis, then, the total cost of the system for each mix i (i=A,B,...,G) can be calculated from the formula:

$$Y_i = L_i + B_1X_1 + B_2X_2 + \ldots + B_6X_6 \tag{15}$$

where L_i is the monthly lease cost of line mix i, X_j is the number of toll calls to Zone j per month, and B_j is the average cost of a toll call to Zone j (see Table 4). In the case of Line Mix A, this formula would read:

$$Y_A = \$21{,}076 + (\$2.50)X_1 + (\$2.80)X_2 + \ldots + (\$3.60)X_6 \tag{16}$$

The total cost of each line mix would change for each maximum delay time value due to the different values of X_j that would be generated.

The value of L for each mix is calculated from the formula

$$L = n_1C_1 + n_2C_2 + n_3C_3 + n_4C_4 + n_5C_5 + n_6C_6 \tag{17}$$

where n_k is the number of lines in each zone to be leased, and C_k is the cost per line including federal tax. Table 3 lists these cost figures.

Experimental Results

A summary of the number of toll calls generated by zone over the three day simulation is given in Table 6. By applying the cost formula [Equation (15)] to the projected monthly toll call figures, the total monthly costs of each line mix for both maximum delay time variables have been calculated. These results appear in Table 7, and are also illustrated in ascending order of cost in Figure 7.

Table 6. Number of Toll Calls

Zone	A	B	C	D	E	F	G
MDT (15)							
1	15	45	34	21	17	37	18
2	22	64	50	26	17	46	51
3	25	55	56	15	0	17	22
4	57	1	6	12	9	40	1
5	0	2	2	4	15	15	12
6	0	17	9	22	50	34	46
MDT (20)							
1	9	27	35	14	12	24	15
2	24	40	40	23	22	64	17
3	32	43	52	11	1	14	9
4	72	0	2	23	5	26	5
5	0	2	12	1	9	16	14
6	0	0	11	6	23	36	31

The base of comparison is the line mix A, the mix employed by the company at the time of this study. For the 15 minute MDT set, only mix B shows a higher total cost than mix A, and for the 20 minute MDT set, mix A represents the highest total cost. The mix F is slightly less costly than mix D for the 15 minute MDT; however, its toll call costs are very high which indicates that the mix is not sufficient to handle the volume of traffic. On the other hand, mix D exhibits a relatively low total toll cost figure in both the MDT alternatives. In this case, the total monthly charge could be expected to remain closer to the fixed lease charge with smaller variation. Further, the system would not rely so heavily on toll calls which are much more expensive in terms of cost per minute than WATS lines (see Table 3). On the basis of cost alone, then, the mix D appears to be the best choice for both MDT alternatives. For the 15 minute MDT, the savings over mix A would be approximately $1770 per month, or $21,240 annually. The savings for the 20 minute MDT would be $2739 monthly. Curiously, the toll costs for mix A increased with the increased MDT, while the cost decreased for all the other mix alternatives.

Table 7. Monthly Cost Figures by Line Mix

		TOLL COSTS*		TOTAL COST	
Mix	Lease Cost*	MDT (15)	MDT (20)	MDT (15)	MDT (20)
A (000506)	$21,076	$2,594	$3,013	$23,670	$24,089
B (002504)	20,075	4,014	2,389	24,089	22,464
C (003404)	19,965	3,413	3,324	23,378	23,289
D (022304)	19,635	2,269	1,714	21,904	21,350
E (023403)	20,845	2,627	1,685	23,472	22,530
F (022303)	17,555	4,275	4,104	21,830	21,658
G (022403)	19,250	3,512	2,150	22,762	21,400

*Includes 10% Federal Excise Tax.

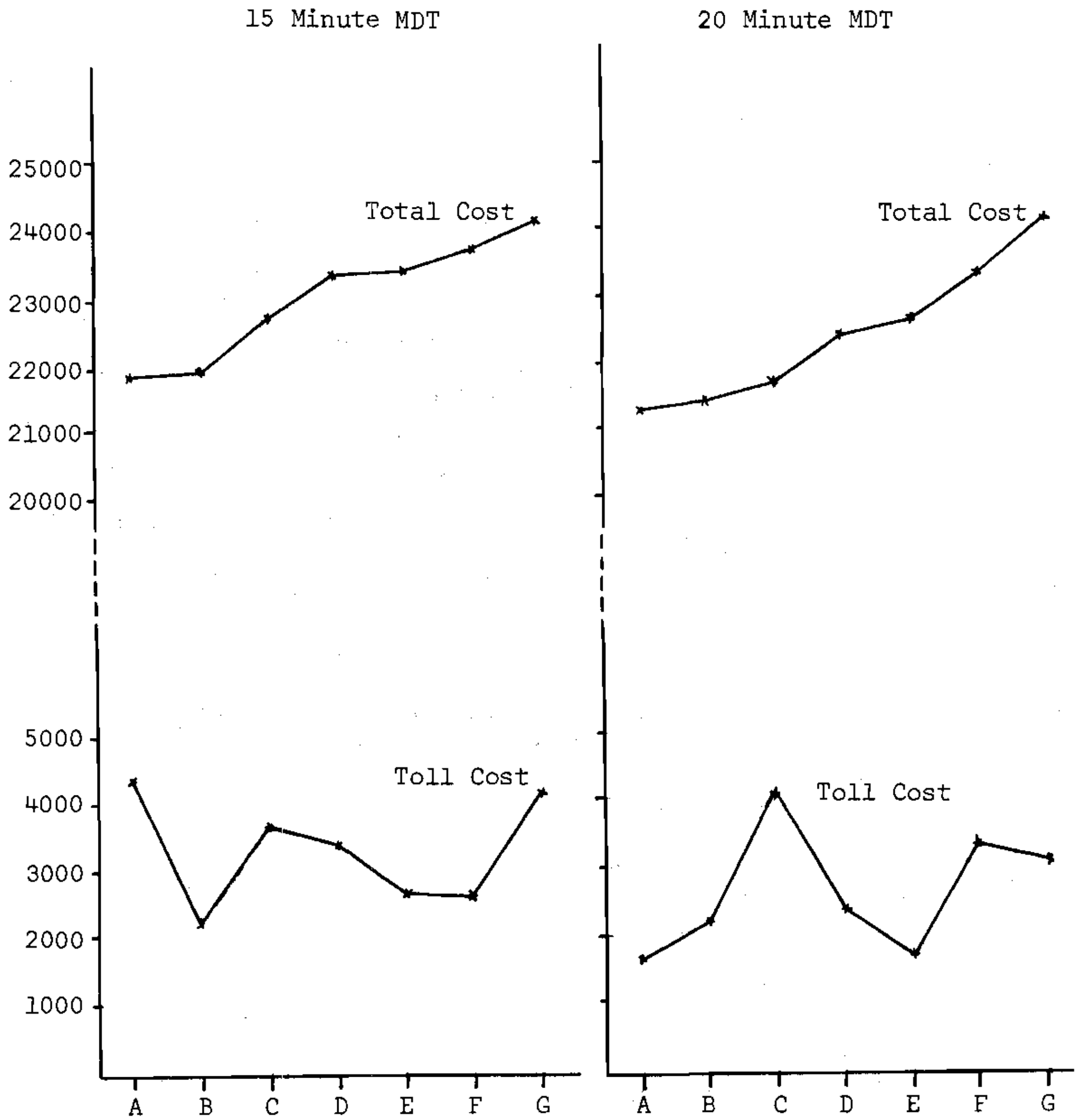

Figure 7. Cost Figures of Each Mix in Ascending Order

A comparison of the variables describing the serviceability components of the mixes revealed that mix D shows a low average delay time in the line queues. Referring to Table 8, the average time in queue over all the zones, MDT (15), is 11.16 time units, or 5.6 minutes. Although mix E shows a 5.0 minute average, the closest practicable competitor is mix G which shows an average of 6.9 minutes in queue. These average times are supported by the percentage of zero entries experienced at each "storage" of zone lines (Table 9). Without considering the influence of zero entries on the average time spent in queue, the waiting time still remains low, the average increase being 1.81 minutes. In both the In Queue Time averages, the increased waiting time for the 20 minute MDT alternative is approximately 1.75 overall.

Under the maximum delay time criteria for usage, it appears best to try for relatively equal periods of delay for all the zones. From Table 8, mix D shows 8.8, 5.5, 4.0, and 4.1 minutes average time in queue for Zones 2, 3, 4, and 6, respectively, MDT (15), while mix G shows 9.9, 5.8, 2.3, and 7.7 minutes and mix F, 9.9, 4.4, 5.1, and 7.8 minutes. The other mixes are much more erratic. The time spent in queue has been further described by the percentages of call requests that were handled within increasing five-minute intervals, as reported in Tables 10 and 11. Once again mix D exhibits relatively even percentages, with the exception of the Zone 2 line calls. However, since the simulator generated 4.1 per cent more Zone 1 calls than might actually be expected, the load on Zone 2 lines probably should be less. Tables 10 and 11 also indicate the percentages of calls that were not

Table 8. Average Time in Queue: All Entries*

	MIX													
	A		B		C		D		E		F		G	
Zone	15	20	15	20	15	20	15	20	15	20	15	20	15	20
2							17.51	23.51	17.92	25.12	19.91	26.67	19.80	22.14
3			25.68	33.23	22.62	31.13	10.89	14.90	3.06	4.11	8.82	12.77	11.68	14.31
4	22.34	30.40	3.28	2.03	5.32	4.27	8.00	11.65	3.87	6.12	10.34	3.18	4.59	4.77
6	1.32	2.19	8.62	9.36	6.47	10.95	8.16	7.60	14.94	15.85	15.66	19.99	14.28	18.62
Average	5.81	8.15	6.26	7.44	5.74	7.72	5.58	7.20	4.96	6.40	6.84	9.06	6.04	7.45

*Units are in half-minutes.

Table 9. Percentage of Zero Entries

	A		B		C		D		E		F		G	
Zone	15	20	15	20	15	20	15	20	15	20	15	20	15	20
2							10.82	8.51	12.14	5.81	11.77	5.79	9.94	12.76
3			2.06	2.71	5.39	2.09	28.61	27.30	63.27	66.75	38.50	26.00	29.21	25.85
4	3.94	1.52	62.98	68.89	50.70	54.42	31.79	36.24	58.74	52.71	33.68	30.03	48.91	57.87
6	72.26	67.29	32.91	33.38	42.22	32.82	35.63	40.97	13.82	19.67	13.37	14.55	18.96	19.39

Table 10. Percentages of Calls Handled Within Time Intervals

15 Minute MDT

Zone	A (000506)	B (002504)	C (003404)	D (022304)	E (023403)	F (022303)	G (022403)
Within 0-5 Minutes							
2				29.90	30.57	25.21	24.77
3		7.64	13.99	56.74	88.05	65.12	51.49
4	12.18	87.02	77.22	66.61	85.02	59.53	82.55
6	97.63	67.52	75.32	64.75	34.56	33.43	38.90
Within 5-10 Minutes							
2				23.88	17.36	16.92	16.82
3		11.36	14.27	20.33	9.07	16.28	22.52
4	17.13	9.13	15.44	21.73	9.69	16.69	10.44
6	2.23	12.91	13.46	20.31	28.63	27.17	26.40
Within 10-15 Minutes							
2				24.23	27.50	26.70	28.75
3		30.85	38.66	11.82	2.88	10.85	14.36
4	47.34	3.37	5.62	7.99	3.96	14.77	6.54
6	.15	14.89	8.42	11.49	25.11	29.59	21.91
Within 15-20 Minutes			*TOLL CALLS AND HIGHER LINES*				
2				21.99	24.58	31.18	29.66
3		50.15	33.08	11.11	0	7.75	11.63
4	23.34	.48	1.72	3.67	1.32	9.01	.47
6	0	4.68	2.81	3.45	11.71	9.82	12.78

Table 11. Percentage of Calls Completed Within Time Intervals

20 Minute MDT

Zone	A (000506)	B (002504)	C (003404)	D (022304)	E (023403)	F (022303)	G (022403)
0-5 Minutes							
2				19.48	19.47	16.01	30.05
3		7.84	8.63	45.91	86.23	53.75	46.14
4	5.53	93.32	84.07	60.63	76.36	51.70	81.33
6	91.22	61.58	60.42	68.99	42.18	31.67	36.30
5-10 Minutes							
2				18.33	14.44	15.24	13.73
3		6.13	6.99	19.11	5.19	21.75	20.05
4	9.78	4.23	10.88	14.00	12.05	19.35	8.80
6	8.38	19.75	13.66	17.78	17.44	17.26	16.76
10-15 Minutes							
2				24.88	18.68	18.14	16.32
3		8.64	17.26	13.40	5.45	8.00	16.91
4	23.09	2.44	2.68	9.23	7.08	10.22	7.25
6	.40	13.49	12.11	8.82	22.21	18.97	17.93
15-20 Minutes				*TOLL CALLS AND HIGHER LINES*			
2				20.29	25.27	25.76	19.87
3		33.07	37.87	14.14	2.86	9.50	11.35
4	41.37	0	2.05	9.23	3.46	12.85	1.85
6	0	3.27	10.00	3.41	12.82	21.11	18.66
20-25 Minutes							
2				17.02	22.14	24.85	20.03
3		44.32	29.25	7.44	.26	7.00	5.56
4	20.23	0	.32	6.92	1.05	5.88	.77
6	0	1.91	3.80	1.00	5.37	10.98	10.35

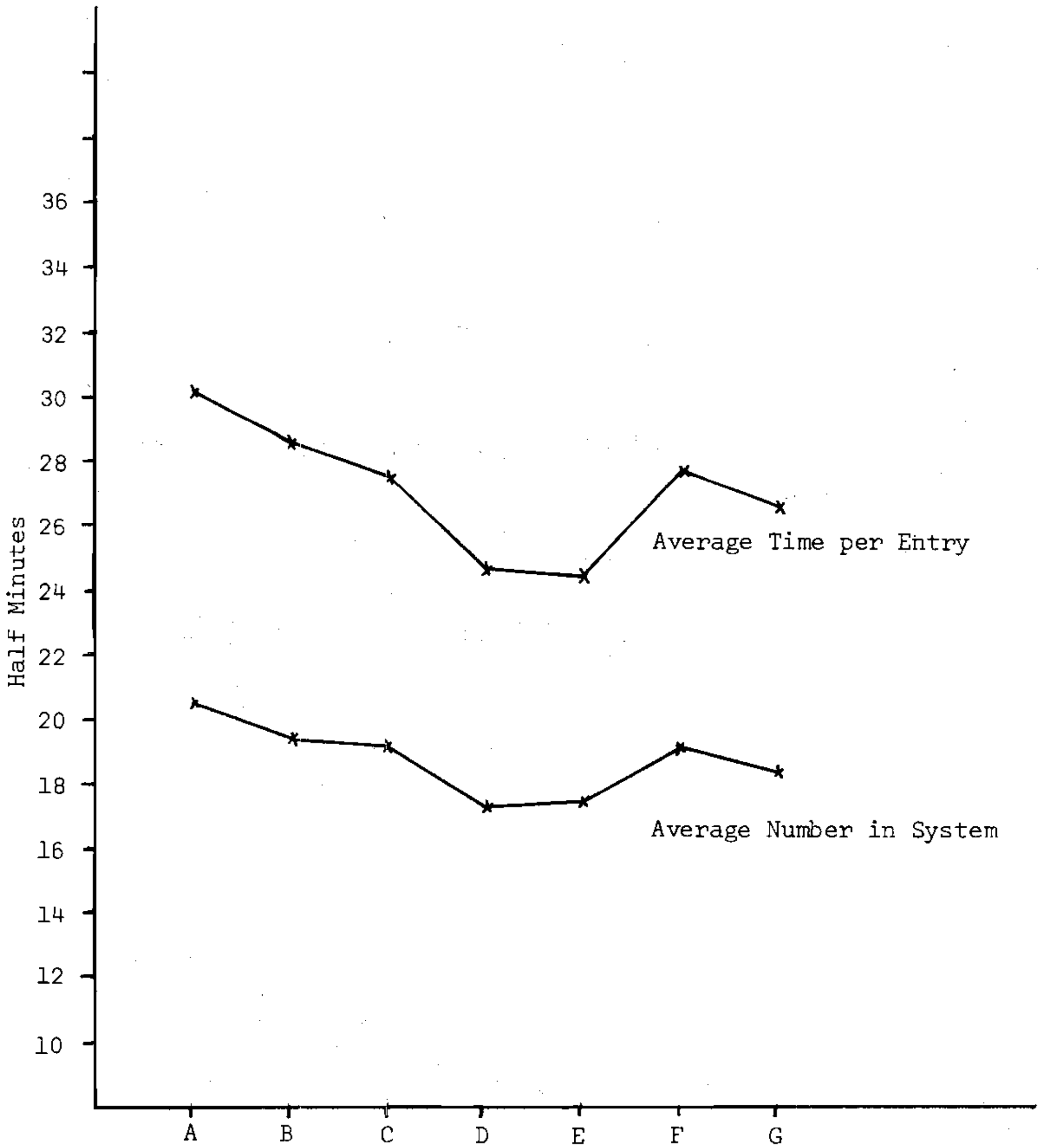

Figure 8. Average Time per Entry/Average Number in System
MDT = 15 Minutes

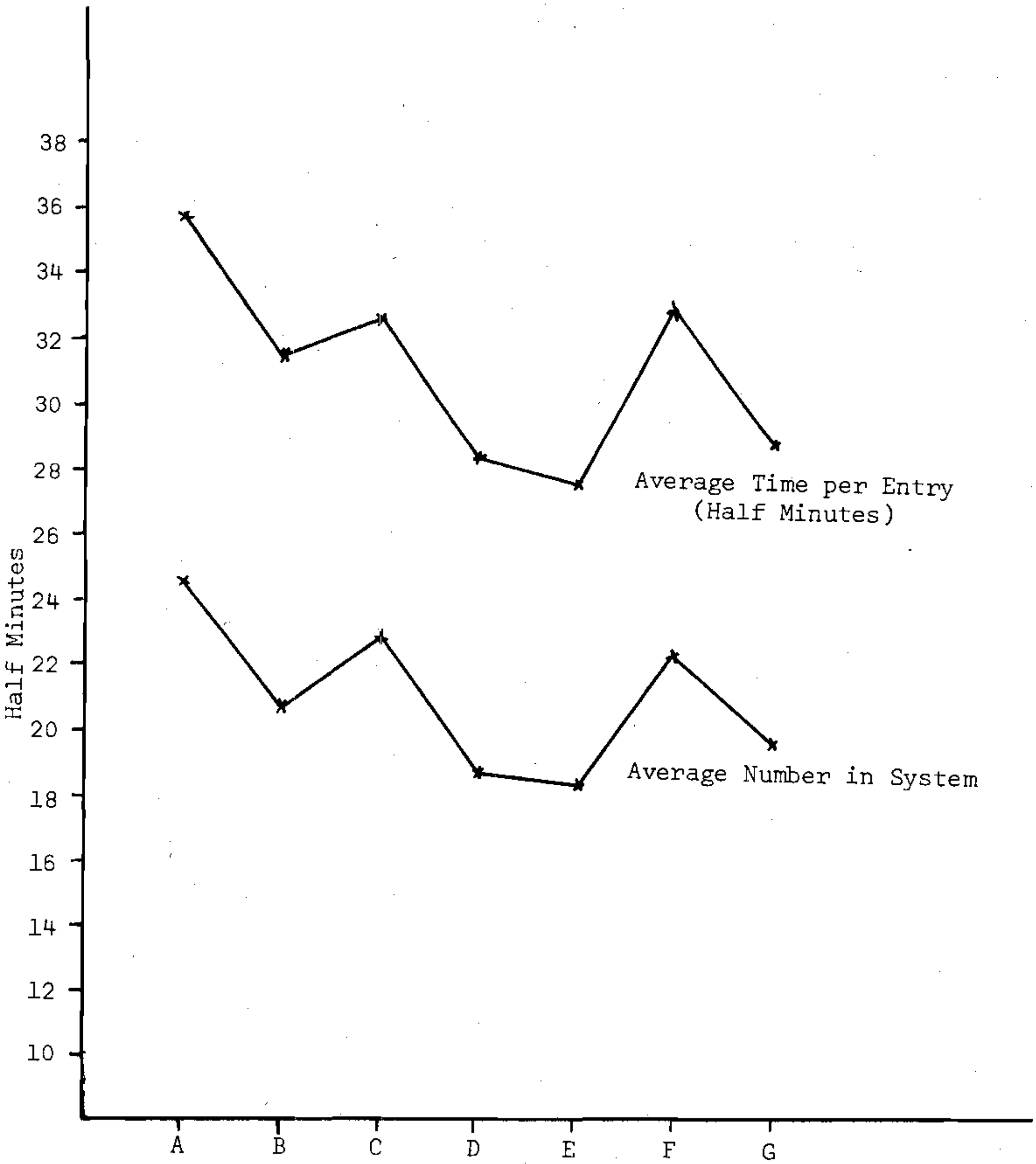

Figure 9. Average Time per Entry/Average Number in System
MDT = 20 Minutes

able to access their proper line within the maximum delay time. Again, mix D shows a smoother proportion than the other mixes as well as lower total percentages than mixes G and F.

The Average Contents of Queues (Table 12) is another indicator of expected time in queue. It also serves to point out differences in the two maximum delay time alternatives, i.e. generally more calls are waiting to be handled at one time with the higher MDT. Figures 8 and 9 show another dimension of the relationship between the numbers of tickets in the system and the average time spent in the system, including the service time, for both MDT's. Here, too, the variables of mix D show it to be consistent with a good service system.

Table 12. Average Contents of Queues

Zone	A (000506)	B (002504)	C (003404)	D (022304)	E (023403)	F (022303)	G (022403)
15 Minutes							
2				3.07	3.50	3.60	3.84
3		7.89	7.16	1.39	.41	1.02	1.40
4	11.31	.62	1.02	1.51	.79	2.10	.87
6	.27	1.83	1.38	1.92	3.17	2.31	3.02
20 Minutes							
2				4.31	4.66	5.08	4.08
3		9.75	10.30	1.80	.46	1.48	1.76
4	14.93	.37	.81	2.12	1.18	2.47	.92
6	.49	2.03	2.34	1.60	3.10	4.07	3.81

One of the most useful variables in describing the system is that of Average Utilization (see Table 13). These figures include the use of lines by lower rated calls. The lowest rated zones for all the mixes consistently exceed the 85 per cent mark of highest acceptable usage, meaning that few calls would experience no delay. However, the utilization factors of the next higher lines could easily accommodate some of that traffic, particularly for mixes D, E, F, and G. By altering the usage policy so that Zone 1 and 2 calls can be spread to Zone 3 lines before reaching the MDT, these utilization factors might be closely aligned. Mix D shows higher factors of utilization than does mix G, which is manifested in economy of operation.

Table 13. Average Utilization

Zone	A (000506)	B (002504)	C (003404)	D (022304)	E (023403)	F (022303)	G (022403)
15 Minutes							
2				.9224	.9178	.9098	.9135
3		.9773	.9617	.7834	.5422	.7181	.7724
4	.9711	.6620	.7267	.8296	.6682	.8226	.7144
6	.6024	.7808	.7580	.7424	.8087	.8419	.8021
20 Minutes							
2				.9416	.9178	.9409	.9114
3		.9580	.9836	.7836	.5422	.8042	.8228
4	.9778	.6335	.6943	.7727	.6682	.7747	.6701
6	.6201	.7986	.7802	.7126	.8072	.7934	.8012

CHAPTER V

CONCLUSIONS AND RECOMMENDATIONS

Conclusions

The problem of determining the most productive mix of WATS lines has been separated into investigating two models in relation to each other--a mathematical model and a cost model. The mathematical model is treated in the simulation program and seeks primarily to determine the effect of various line mixes on the number of toll calls generated therefrom and the relative utilization of the different zone lines. The cost model takes the information on toll call occurrences developed by the simulation program, computes the cost associated with the toll calls and adds to it the fixed cost of the respective WATS line mix. Although cost considerations are of prime importance, it is necessary to evaluate the system on the basis of line utilization, acceptable waiting time limits, percentage of toll call spill-over, and average time a call remains in the system. In the preceding chapter these various criteria were set forth for comparison.

The results of the simulation program and the corresponding cost model point to the recommendation of line mix D (022304). This recommendation holds for both maximum delay times of 15 and 20 minutes. The decision of which MDT to adopt rests with the company; they feel that there is a definite cost associated with delay in obtaining a line. The difference in cost between MDT(15) and MDT(20) is projected at $554.00

per month, or $6,648 annually. The average delay time varies less than two minutes between the two maximum delay times (5.6 minutes versus 7.4 minutes). It is recommended that the 15 minute MDT be tried initially in order to ascertain the actual usage of toll facilities and, if necessary, increase the maximum delay time to as much as 20 minutes to reduce toll call costs. The important factor in making any system workable is to be sure the users are well informed of the usage procedures and adhere to them.

The Direct Access feature of the simulator was activated; however, the number of calls that managed to access lines was too small to show any effect. Later information from the company indicated that approximately 75 Direct Access calls were experienced every day. Time did not allow alteration of the program statements to accurately simulate this number of calls. Nevertheless, recognizing that 75 calls represent over 10 per cent of the total number of calls attempted daily, it is believed that such direct access feature should be at least curtailed. Since a minimal delay will be encountered on most calls with the MDT system, the advantages and importance of the direct access feature could well be reserved for after hours calls only.

Recommendations

Recent information has indicated that the calling patterns at this company have shifted somewhat. A sampling[†] of call tickets from

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[†]The sample was composed of call tickets for nine days which were separated and counted.

the month of September, 1972, revealed the following percentages of traffic to the six WATS zones:

Table 14. Current Percentages of Traffic to WATS Zones

Zone	Percentage
1	6.2
2	19.3
3	26.7
4	26.6
5	5.8
6	15.5

In comparison with Table 4, most noticeable are the changes in volume of traffic to Zones 3 and 4. This shift in traffic flow supports the case for periodic review of the telecommunications system, perhaps annually. For example, the percentages above point toward the adoption of a line mix such as mix G (022403) or E (023403) with earlier spill-over into Zone 3 and Zone 4 lines.

Since inception of this study, the company has changed to a line mix of (002404) at a monthly cost of $18,370. This is accompanied by some $3600 in toll bills. In order to test the true effectiveness of this mix, the program would need to be altered to provide almost immediate spilling over of call requests from Zone 3 lines to Zone 4, and Zone

4 to Zone 6 rather than requiring the delay of 15 or 20 minutes. Another change, which is being experienced nationwide, is an increase in average duration of calls. The recent toll bill reporting 1000 toll calls at a cost of $3600 was sampled, and the average call was found to last ten minutes. Better use of WATS and lower toll costs could be effected by educating the users in efficient telephone communication. For example, telephone calls should be planned before they are placed, and the caller should be aware of his telephone voice to the point that everything he says will be distinct and understandable.

This company was hesitant about increasing the number of zones that lines would be leased from because it would increase the "look-up" time spent by the operator. It is suggested that the callers be informed of the WATS zone their calls are routed to, and that they record this to inform the operator on later calls. Also, if it does not already exist, a list of the area codes and which WATS zone they fall under should be available to the operator.

Due to the type of data available from the company, the program was somewhat limited in how closely it could represent the different events in telecommunications usage. In this case, much of the data had been collected before the simulator was designed. The greatest problem was in accurately defining and understanding the already collected data, and in knowing what had been included or not. The solution here would be for the author to detail what data should be collected and how it is to be recorded. Additional or detailed information that would have been quite useful includes the percentage of calls that reached busy signals, broken down by zone and time of day, and how long such calls were delayed before finally being successfully completed.

It would also be helpful to have larger samples of calls on which to base their distributional properties. For example, the relatively small number of calls directed to Zones 1 and 5 in the sample was insufficient to determine the interarrival distribution by the hour. During some hours the calls seemed more to be uniformly distributed than exponentially distributed, perhaps because the number of calls sampled was fewer than 15. This in turn threatens the accuracy of call request generation. Another essential statistic is the number and distribution of calls which were "unsuccessful," i.e. the called party was not available, thus requiring a retry or return call. Such calls significantly affected the distribution of the holding times data, but the actual number of these calls was not recorded. This information would probably have to come from the callers rather than the operators.

Some changes in the program would help in extracting information. Simulating three days in each run instead of one day necessitated modifications in the program whose effects were not immediately recognized and adjusted. In tabulating the results for three consecutive "days," the clock time was used to divide the intervals into one-hour periods. However, the time simulated after the switchboard closed and before 8:00 a.m. the next day caused the hours (9, 10, 11, ..., o'clock) of the day to fall between the upper and lower limits of each interval in the tables. Variable 10 modified by a multiple indicating succeeding days could be used to separate the tabulated values by the hours of the day.

Most significant was the error in the offset times for Zones 5 and 6 (10:00 a.m. and 11:00 a.m., respectively). The result was that calls for these areas were generated at the beginning of the day for days two and three. To correct for this, any toll calls generated in those zones during the time before those zones "open up" were subtracted from the total. Otherwise, the net effect should not significantly alter the important decision variables. The program GENERATE statements for Zones 5 and 6 should be changed to read

```
5    Generate    V11
6    Generate    V12
```

where V11 = X19 + 240 and V12 = X19 + 360. The value of X19 is the clock time C1 at the beginning of each succeeding day.

Each run consumes between 1.38 minutes and 2.43 minutes depending on the complexity of the line mixes. The value of sampling the SAVEX Cells 11-16 which indicate how many calls before each natural zone are waiting to be served is negligible and its omission might reduce the computer time used.

The program can be easily changed to accommodate different inter-arrival and holding times, particularly if the exponential is maintained. Changes in the systems usage policy can be effected by creating opportunities for shifting call requests to higher rated lines prior to reaching the MDT. Probably the easiest method would be to compare queue lengths between zones and to shift requests when the difference in queue length reaches a certain point. This could result in slightly higher utilization of the upper 2-3 zone lines. Such a change might also have a beneficial effect on the volume of toll calls generated.

Very recent information has indicated that WATS rates will rise approximately 2 per cent soon. A rise in toll rates is also expected. Again this emphasizes the importance and value of periodically evaluating and perhaps changing the telecommunications system in order to enjoy peak efficiency and service without sacrificing economy.

APPENDIX A

```
 LOC  NAME          X     Y     Z     SEL   NBA   NBB   MEAN  MOD   REMARKS     F

      JOB

 *    WATS SYSTEM SIMULATION - TICE THESIS   1972

 *    WATS LINE MIX D (022304)

 10   VARIABLE      C1-X19
 11   VARIABLE      X19+240
 12   VARIABLE      X19+360
 10   FUNCTION      RN1   C24         EXPONENTIAL FUNCTION
0     0     .1      .104  .2    .222  .3    .355  .4    .509  .5    .69
.6    .915  .7      1.2   .75   1.38  .8    1.6   .84   1.83  .88   2.12
.9    2.3   .92     2.52  .94   2.81  .95   2.99  .96   3.2   .97   3.5
.98   3.9   .99     4.6   .995  5.3   .998  6.2   .999  7     .9997 8

 21   FUNCTION      V10   D5          I A T  ZONE 1
120   14    480     12    840   16    960   14    1080  16
 22   FUNCTION      V10   D6          I A T  ZONE 2
120   10    240     7     360   10    600   12    720   8     1080  10
 23   FUNCTION      V10   D6          I A T  ZONE 3
120   9     480     7     720   13    840   11    960   9     1080  8
 24   FUNCTION      V10   D6          I A T  ZONE 4
360   5     600     7     720   5     840   7     960   5     1080  6
 25   FUNCTION      V10   D5          I A T  ZONE 5
480   16    600     14    720   21    840   14    1080  22
 26   FUNCTION      V10   D4          I A T  ZONE 6
600   6     840     7     960   6     1080  7
 29   FUNCTION      RN1   D3          FUNCTION TO ASSIGN PRIORITIES--DIRECT ACCESS
0.93  0     0.98    12    1.0   24

 1    CAPACITY      1                 NO. OF ZONE 1 LINES
 2    CAPACITY      2                 NO. OF ZONE 2 LINES
 3    CAPACITY      2                 NO. OF ZONE 3 LINES
 4    CAPACITY      3                 NO. OF ZONE 4 LINES
 5    CAPACITY      1                 NO. OF ZONE 5 LINES
 6    CAPACITY      4                 NO. OF ZONE 6 LINES
 10   CAPACITY      1000

 1    TABLE         C1    120   120   50          P5 LINE 1 CALLS
 2    TABLE         C1    120   120   50          P5 LINE 2 CALLS
 3    TABLE         C1    120   120   50          P5 LINE 3 CALLS
 4    TABLE         C1    120   120   50          P5 LINE 4 CALLS
 5    TABLE         C1    120   120   50          P5 LINE 5 CALLS
 6    TABLE         C1    120   120   50          P5 LINE 6 CALLS
 7    TABLE         C1    120   120   50          NO. CALLS PER HOUR DIRECT ACCE
 8    TABLE         C1    60    60    6        NO. CALLS/30 MIN.LINE 6 PRE-11AM
 9    TABLE         C1    120   120   50          CALLS/HR. GOING ON HIGHER LINE
11    TABLE         C1    120   120   50       TOLL CALLS ZONE 1  HOUR
12    TABLE         C1    120   120   50       TOLL CALLS ZONE 2  HOUR
```

```
13    TABLE      C1    120   120   50      TOLL CALLS ZONE 3  HOUR
14    TABLE      C1    120   120   50      TOLL CALLS ZONE 4  HOUR
15    TABLE      C1    120   120   50      TOLL CALLS ZONE 5  HOUR
16    TABLE      C1    120   120   50      TOLL CALLS ZONE 6  HOUR
22    QTABLE     2     10    10    12      DELAY TIME, QUEUE P5=2
23    QTABLE     3     10    10    12      DELAY TIME, QUEUE P5=3
24    QTABLE     4     10    10    12      DELAY TIME, QUEUE P5=4
26    QTABLE     6     10    10    12      DELAY TIME, QUEUE P5=6
* BLOCKS 1 THROUGH 76 GENERATE CALL REQUESTS
1     GENERATE   0                     11                    ZONE 1
2     GENERATE   0                     12                    ZONE 2
3     GENERATE   0                     13                    ZONE 3
4     GENERATE   0                     14                    ZONE 4
5     GENERATE   V11                   15                    ZONE 5
6     GENERATE   V12                   16                    ZONE 6
11    SEIZE      11                    21                    DUMMY FACILITY
12    SEIZE      12                    22                    DUMMY FACILITY
13    SEIZE      13                    23                    DUMMY FACILITY
14    SEIZE      14                    24                    DUMMY FACILITY
15    SEIZE      15                    25                    DUMMY FACILITY
16    SEIZE      16                    26                    DUMMY FACILITY
21    ASSIGN     2     FN21            31          *2    FN10  I A T
22    ASSIGN     2     FN22            32          *2    FN10  I A T
23    ASSIGN     2     FN23            33          *2    FN10  I A T
24    ASSIGN     2     FN24            34          *2    FN10  I A T
25    ASSIGN     2     FN25            35          *2    FN10  I A T
26    ASSIGN     2     FN26            36          *2    FN10  I A T
31    ASSIGN     2     K11             41                    P2 = REAL ZONE
32    ASSIGN     2     K12             42                    P2 = REAL ZONE
33    ASSIGN     2     K13             43                    P2 = REAL ZONE
34    ASSIGN     2     K14             44                    P2 = REAL ZONE
35    ASSIGN     2     K15             45                    P2 = REAL ZONE
36    ASSIGN     2     K16             46                    P2 = REAL ZONE
41    ASSIGN     5     K2              51                    P5=ASSIGN LINE
42    ASSIGN     5     K2              52                    P5=ASSIGN LINE
43    ASSIGN     5     K3              53                    P5=ASSIGN LINE
44    ASSIGN     5     K4              54                    P5=ASSIGN LINE
45    ASSIGN     5     K6              55                    P5=ASSIGN LINE
46    ASSIGN     5     K6              56                    P5=ASSIGN LINE
51    GATE       NU10                  71
52    GATE       NU10                  72
53    GATE       NU10                  73
54    GATE       NU10                  74
55    GATE       NU10                  75
56    GATE       NU10                  76
71    RELEASE    11                    91
72    RELEASE    12                    91
73    RELEASE    13                    91
74    RELEASE    14                    91
75    RELEASE    15                    91
76    RELEASE    16                    91
```

```
* BLOCKS TO TEST DIRECT ACCESS
 81     ASSIGN       3      FN11                    82
 82     PRIORITY     *3                             92
 93     COMPARE      P3     G5     K12     .400     94     140
 94     ADVANCE                            BOTH     96     140
 96     STORE        *5                             97            14     FN10
 97     TABULATE     7                              450

 91     ENTER        10                             92
 92     MARK                                        140
 140    SAVEX        *2+    K1                      400
* QUEUES IN FRONT OF ASSIGNED LINES--P5      DELAY TIME TABLED
 400    QUEUE        *5                    ALL      149    151
 149    COMPARE      V10    GE     K1080            450
 150    GATE         SNF*5                 .04      155    157                  P5 LINE FREE
 155    SAVEX        *2-    K1                      160

 160    ENTER        *5                             165
 165    ADVANCE                                     166           14     FN10
 166    LEAVE        *5                             301
 301    TABULATE     *5                             450
 450    LEAVE        10                             98

 157    ASSIGN       4+     K1                      158
 158    PRIORITY     5      BUFFER         BOTH     159    400
 159    COMPARE      P4     E      K4               450

* DELAY TIME REACHES MAXIMUM
 151    COMPARE      D1     G      K30              220
 220    ADVANCE                            BOTH     221    229
 221    COMPARE      V10    LE     K360    BOTH     250    229
 250    GATE         SNF6                  .06      252    157
 252    SAVEX        *2-    K1                      255
 255    STORE        6                              303           14     FN10
 303    TABULATE     8                              450

 229    ASSIGN       7      P5                      230
 230    INDEX        7      1              ALL      240    242                  TRY NEXT ZONE
 240    GATE         SNF*1                 .06      260    157
 260    ENTER        *1                             261
 261    SAVEX        *2-    K1                      263
 263    ADVANCE                                     265           14     FN10
 265    LEAVE        *1                             304
 304    TABULATE     9                              450

 241    COMPARE      P1     L      K6               243                         EXHAUST LINES
 243    ASSIGN       7      P1                      244
 244    PRIORITY     10                             230
* CALLS ARE PLACED ON TOLL LINES AFTER MAXIMUM DELAY
 242    ADVANCE                            .06      269    157                  GO TOLL
 269    SAVEX        *2-    K1                      270
```

```
 270    ADVANCE                                  305          14     FN10
 305    TABULATE     *2                          450

 98     TERMINATE

 350    GENERATE    1080   1      110            281
 281    SAVEX       11     K0                    282
 282    SAVEX       12     K0                    283
 283    SAVEX       13     K0                    284
 284    SAVEX       14     K0                    285
 285    SAVEX       15     K0                    286
 286    SAVEX       16     K0                    351
 351    SEIZE       10                           352
 352    GATE        SE10                         353
 353    SAVEX       19     C1                    354
 354    SAVEX       30+    K1           BOTH     355    359
 355    COMPARE     X30    L      K3             356
 356    RELEASE     10                           357
 357    ADVANCE                                  281          1080
 359    TERMINATE   R

* SAMPLE SAVEX *2 EVERY 15 MINUTES
 360    ORIGINATE   30            50             362          30
 362    PRINT       X11    16                    363
 363    TERMINATE
*
 371    GENERATE    0      1      100            381
 372    GENERATE    0      1      100            382
 373    GENERATE    0      1      100            383
 374    GENERATE    0      1      100            384
 375    GENERATE    0      1      100            385
 376    GENERATE    0      1      100            386
 381    STORE       1                            98           5500
 382    STORE       2                            98           0
 383    STORE       3                            98           0
 384    STORE       4                            98           0
 385    STORE       5                            98           5500
 386    STORE       6                            98           0
*
        START       1
```

APPENDIX B

CHI-SQUARE GOODNESS-OF-FIT TEST

Applied to Interarrival Times by Zone

In each zone the distribution of the interarrival times data is being tested for its conformity to the exponential distribution.

The degrees of freedom are (k-2) where k is the number of class intervals. The expected values were calculated from $g(x) = \hat{\lambda}e^{-\hat{\lambda}x}$ where $\hat{\lambda}$ is set equal to $1/\bar{X}$ and $\bar{X}$ is the average of the observed values.

In all cases, the last class interval is composed of those interarrival times whose expected values are less than 3.00, as suggested by Ostle (10, p.127).

ZONE I	Number of calls observed:	130
	Number of class intervals:	14
	length of interval:	1 minute (1-13)
		17 minutes (14)
	Average arrival rate $\hat{\lambda}$:	1/7.4 minutes = .135
	X^2:	18.53
	$X^2_{.95(12)}$:	21.0
ZONE II	Number of calls observed:	348
	Number of class intervals:	16
	length of interval:	1 minute (1-15)
		11 minutes (16)
	Average arrival rate $\hat{\lambda}$:	1/4.98 minutes = .201
	X^2:	19.25
	$X^2_{.90(14)}$:	21.1

ZONE III

Number of calls observed: 453

Number of class intervals: 15
length of interval: 1 minute (1-14)
9 minutes (15)

Average arrival rate $\hat{\lambda}$: 1/4.17 minutes = .240

X^2: 22.43

$X^2_{.95(13)}$: 22.4

$X^2_{.975(13)}$: 24.7

ZONE IV

Number of calls observed: 608

Number of class intervals: 11
length of interval: 1 minute (1-10)
5 minutes (11)

Average arrival rate $\hat{\lambda}$: 1/2.64 minutes = .379

X^2: 15.88

$X^2_{.95(9)}$: 16.9

ZONE V

Number of calls observed: 141

Number of class intervals: 13
length of interval: 1 minute (1-12)
16 minutes (13)

Average arrival rate $\hat{\lambda}$: 1/7.16 minutes = .140

X^2: 20.57

$X^2_{.975(11)}$: 21.9

ZONE VI

Number of calls observed: 309

Number of class intervals: 12
length of interval: 1 minute (1-11)
5 minutes (12)

Average arrival rate $\hat{\lambda}$: 1/3.23 minutes = .310

X^2: 19.62 $X^2_{.975(10)}$: 20.5

Summary

The interarrival time data for each WATS zone over the whole day conformed to within a level of $\alpha = .05$ to the exponential distribution as tested by the Chi-Square Goodness-of-Fit Test.

Applied to Holding Times Data

The distribution of the holding times data as compiled on a whole day basis is being tested for its conformity to the exponential distribution.

The expected values were calculated from $g(x) = \bar{u}\, e^{-\bar{u}x}$ where $\bar{u}$ is set equal to $1/\bar{x}$ and $\bar{x}$ is the average holding time of the observed values.

The degrees of freedom are (k-2) where k is the number of class intervals. Except for the last class interval, the length of each interval is one minute. The last interval is composed of calls whose holding times were between 30 and 45 minutes.

Number of calls observed:	2060
Number of class intervals:	30
Average holding time $\bar{u}$:	1/6.47 minutes = .155
X^2:	162.24
$X^2_{.995(28)}$:	51.0

Discussion

Major deviations from the exponential are found at the one and two minute intervals. Causes for these deviations were discussed on page 41. Subtracting their contribution from the X^2 statistic, the remainder conforms to the $X^2_{.995(28)}$ value, 51.0, however, as shown below:

$$
\begin{array}{rl}
162.24 & \\
-\ \underline{93.91} & \text{1 minute interval} \\
68.33 & \\
-\ \underline{17.71} & \text{2 minute interval} \\
50.62 &
\end{array}
$$

APPENDIX C

Calculations for Mean and Variance of Service Times Data

Mean $\bar{x} = \frac{\Sigma fx}{n}$

Variance $s^2 = \frac{(\Sigma fx^2) - (\Sigma fx)^2/n}{n - 1}$

$$\Sigma f = n = 2060$$

$$\Sigma fx = 13{,}327$$

$$\Sigma fx^2 = 171{,}385$$

$$\bar{x} = 6.47$$

$$\bar{x}^2 = 41.86$$

$$s^2 = 41.36$$

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